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UNIVERSITY OF MUMBAI

Syllabus for: F. Y. B. Sc. /F. Y. B. A.
Program: B.Sc. /B.A.
Course: Mathematics

(Credit Based Semester and Grading System with effect from the Academic year (2014–2015)

Revised Syllabus in Mathematics
Credit Based Semester and Grading System
First Year B. Sc. / B. A. 2014-2015

Preamble

Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. It is imperative that the content of undergraduate syllabi of Mathematics should support other branches of science such as Physics, Statistics and Computer Science. The present syllabi of F. Y. B. Sc./ F. Y. B. A. for semester I and II has been designed as per U. G. C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly. There are two courses for 'Calculus ' spread over two semesters. Calculus is applied and needed in every conceivable branch of science. The two courses on, 'Algebra and Linear Algebra ' spread over two semesters develops mathematical reasoning and logical thinking and has applications in science and technology.

SEMESTER I

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/ Week
USMT101 UAMT101	I	Real Number System and Sequence of Real Numbers	3	3
	II	Sequences (contd.)		
	III	Limits and Continuity		
ALGEBRA I				
USMT102	I	Integers and divisibility	3	3
	II	Functions and Equivalence relations		
	III	Polynomials		

SEMESTER II

CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/ Week
USMT201	I	Series	3	3
	II	Continuous functions and Differentiation		
	III	Application of differentiation		

LINEAR ALGEBRA				
USMT202 UAMT201	I	System of Linear Equation and Matrices	3	3
	II	Vector spaces		
	III	Basis and Linear Transformation		

Teaching Pattern

1. Three lectures per week per course.
2. One Tutorial per week per batch per course (The batches to be formed as prescribed by the University).
3. One assignment per course or one project.

F. Y. B. Sc./ F. Y. B. A. Mathematics

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

SEMESTER I

USMT 101/ UAMT 101: CALCULUS I

Brief review: Domain, Range, Types of function: Injective, Surjective, Bijective, Inverse, Composite.

Unit I: Real Number System and Sequence of Real Numbers (15 Lectures)

Real Numbers: Real number system and order properties of R, Absolute value properties AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhood, Hausdorff property, Bounded sets, Continuum property (l.u.b. axiom-statement, g.l.b.) and its consequences, Supremum and infimum, Maximum and minimum, Archimedean property and it's applications.

Sequences: Definition of a sequence and examples, Convergence of sequence, every convergent sequence is bounded, Limit of sequence, uniqueness of limit if it exists, Divergent sequences, Convergence of standard sequences like $c^{1/n}, n^{1/n}, a^n$ etc.

Unit II: Sequences (contd.) (15 Lectures)

Algebra of convergent sequences, Sandwich theorem, Monotone sequences, Monotone convergence theorem and consequences such as convergent of $\left(1+\frac{1}{n}\right)^n$, Subsequences: Definition, Subsequence of a convergent sequence is convergent and converges to the same limit, Cauchy sequence: Definition, every convergent sequence is a Cauchy sequence and converse.

Unit III: Limits and Continuity (15 Lectures)

Limit of Functions: Graphs of some standard functions such as $|x|$, e^x, $\log x$, $\frac{1}{x}$, $ax^2 + bx + c$, $\lfloor x \rfloor$ (Flooring function), $\lceil x \rceil$ (Ceiling function), $x^n (n \geq 3)$, $\sin x$, $\cos x$, $\tan x$, $x \sin \frac{1}{x}$, $x^2 \sin \frac{1}{x}$ over suitable intervals, Limit of a function, evaluation of limit of simple functions using the $\varepsilon - \delta$ definition, uniqueness of limit if it exists, Algebra of limits (with proof), Limit of composite function, Sandwich theorem, Left hand, right hand limits, nonexistence of limits, Limit as $x \to \pm\infty$ and $\lim f(x) = \pm\infty$.

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity.

Recommended Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. Binmore, Mathematical Analysis, Cambridge University Press, 1982.

Additional Reference Books

1. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pte. Ltd.
2. Courant and John, A Introduction to Calculus and Analysis, Springer.
3. Ajit and Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.
5. Ghorpade Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2000.

Assignments (Tutorials)

1. Application based examples of Archimedean property, intervals, neighbourhood.
2. Consequences of continuum property, infimum and supremum of sets.
3. Calculating limits of sequence.
4. Cauchy sequence, monotone sequence.
5. Limit of a function and Sandwich theorem.
6. Continuous and discontinuous functions.

USMT102: ALGEBRA I

Prerequisites:

Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations, Permutations and combinations, nCr, nPr.

Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

Unit I: Integers and divisibility (15 Lectures)
Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal Triangle.

Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a and b and that the g.c.d. can be expressed as $ma + nb,\ m,n \in Z$, Euclidean algorithm, Primes, Euclid's lemma, Fundamental theorem of arithmetic, The set of primes is infinite.

Congruences, definition and elementary properties, Euler's ∅- function, Statements of Euler's theorem, Fermat's theorem and Wilson theorem, Applications.

Unit II: Functions and Equivalence relations (15 Lectures)
Definition of function, domain, codomain and range of a function, composite functions, examples, Direct image $f(A)$ and inverse image $f^{-1}(A)$ of a function $f,$ Injective, surjective, bijective functions, Composite of injective, surjective, bijective functions, Invertible functions, Bijective functions are invertible and conversely, Examples of functions including constant, identity, projection, inclusion, Binary operation as a function, properties, examples.

Equivalence relations, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa, Congruence an equivalence relation on $Z,$ Residue classes, Partition of Z, Addition modulo n , Multiplication modulo n, examples, conjugate classes.

Unit III: Polynomials (15 Lectures)
Definition of polynomial, polynomials over F , where $F = Q, R\ or\ C,$ Algebra of polynomials, degree of polynomial, basic properties, Division algorithm in $F[X]$ (without proof) and g.c.d. of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem, A polynomial of degree n over F has at most n roots. Complex roots of a polynomial in $R[X]$ occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree n in $C[X]$ has exactly n complex roots counted with multiplicity, A non constant polynomial in $R[X]$ can be expressed as a product of linear and quadratic factors in $R[X]$, Necessary condition for a rational number $\frac{p}{q}$ to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is a irrational number where p is a prime number, n^{th} roots of unity, sum of n^{th} root of unity.

Recommended Books

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.
2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.

Recommended Books

1. I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.
2. G. Birkoff and S. Maclane, A Survey of Modern Algebra, Third Edition, Mac Millan, New York, 1965.
3. N. S. Gopalkrishnan, University Algebra, Ne Age International Ltd, Reprint 2013.
4. I .N. Herstein, Topics in Algebra, John Wiley, 2006.
5. P. B. Bhattacharya S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, New Age International, 1994.
6. Kenneth Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition, Mathematics Series.

Assignments (Tutorials)

1. Mathematical induction (The problems done in F.Y.J.C. may be avoided).
2. Division Algorithm and Euclidean algorithm, in Z, Primes and the Fundamental Theorem of Arithmetic.
3. Functions (direct image and inverse image). Injective, surjective, bijective functions, finding inverses of bijective functions.
4. Congruences and Euler's-function, Fermat's little theorem, Euler's theorem and Wilson's theorem.
5. Equivalence relation.
6. Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

SEMESTER II

USMT 201: CALCULUS II

Unit I: Series (15 Lectures)

Series of real numbers, simple examples of series, Sequence of partial sums, convergence of series, convergent and divergent series, Necessary condition: series $\sum a_n$, convergent implies $a_n \to 0$, converse not true, Algebra of convergent series, Cauchy criterion, $\sum \frac{1}{n^p}$ converges for $p > 1$, divergence of $\sum \frac{1}{n}$, Comparison test, limit comparison test, Alternating series, Leibnitz theorem (alternating series test) and convergence of $\sum \frac{(-1)^n}{n}$, Absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test, root test (without proofs) and examples.

Unit II: Continuous functions and Differentiation (15 Lectures)

Properties of Continuous functions: If $f:[a,b] \to R$ is continuous at $x_0 \in [a,b]$ and $f(x_0) > 0$ then there exists a neighbourhood N of x_0 such that $f(x) > 0$ *for all* x *in* N, If $f:[a,b] \to R$ is continuous function then the image $f([a,b])$ is a closed interval, Intermediate value theorem and its applications, Bolzano-Weierstrass theorem (statement only), A continuous function on a closed and bounded interval is bounded and attains its bounds.

Differentiation of real valued function of one variable: Definition of differentiation at a point and on an open set, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, Algebra of differentiable functions, chain rule, Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation (only examples).

Unit III: Application of differentiation (15 Lectures)

Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, Graphing of functions using first and second derivatives, concave, convex functions, points of inflection, Rolle's theorem, Lagrange's and Cauchy's mean value theorems, applications and examples, Monotone increasing and decreasing function, examples, L-hospital rule without proof, examples of indeterminate forms, Taylor's theorem with Lagrange's form of remainder with proof, Taylor polynomial and applications.

Recommended Books

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.
3. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pte. Ltd.

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1. Courant and John, A Introduction to Calculus and Analysis, Springer.
2. Ajit and Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. Ghorpade Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2000.
5. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
6. G. B. Thomas, Calculus, 12th Edition, 2009.

Assignments (Tutorials)

1. Calculating limit of series, Convergence tests.
2. Properties of continuous functions.
3. Differentiability, Higher order derivatives, Leibnitz theorem.

4. Mean value theorems and its applications.
5. Extreme values, increasing and decreasing functions.
6. Applications of Taylor's theorem and Taylors polynomials.

USMT 202/ UAMT 201: LINEAR ALGEBRA

Prerequisites:

Review of vectors in R^2 and R^3 as points, Addition and scalar multiplication of vectors in terms of co-ordinates, Dot product, Scalar triple product, Length (norm) of a vector).

Unit I: System of Linear equations and Matrices (15 Lectures)

Parametric equation of lines and planes, System of homogeneous and non-homogeneous linear equations, the solution of system of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for$(m, n) = (1,2), (1,3), (2,2), (2,3),\ (3,3)$, Definition of n-tuples of real numbers, sum of n -tuples and scalar multiple of n -tuple.

Matrices with real entries, addition, scalar multiplication and multiplication of matrices, Transpose of a matrix, Type of matrices: zero matrix, identity matrix, scalar, diagonal, upper triangular, lower triangular, symmetric, skew-symmetric matrices, Invertible matrices, identities such as $(AB)^t = B^t A^t$,$(AB)^{-1} = B^{-1}A^{-1}$.

System of linear equations in matrix form, elementary row operations, row echelon matrix, Gaussian elimination method, to deduce that the system of m homogeneous linear equations in n unknowns has a non-trivial solution if $m < n$.

Unit II: Vector spaces (15 Lectures)

Definition of real vector space, examples such as R^n with real entries, $R[X]$, space of $m \times n$ matrices over R, space of real valued functions on a non empty set X.

Subspace: Definition, examples of subspaces of R^2 and R^3 such as lines, plane passing through origin, set of 2×2 , 3×3 upper triangular, lower triangular, diagonal, symmetric and skew-symmetric matrices as subspaces of $M_2(R)$, $M_3(R)$, $P_n[x\} = \{a_0 + a_1 x + \dots + a_n x;\ a_i \in R,\ for\ 0 \le i \le n\}$ of $R[X]$, solutions of m homogeneous linear equations in n unknowns as a subspace of R^n; space of continuous real valued functions on a nonempty set X is a subspace of $F(X, R)$, properties of subspace such as necessary and sufficient condition for a non empty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace; union of two subspaces is a subspace if and only if one is a subset of the other, Linear combinations of vectors in a vector space, Linear span $L(S)$ of a non empty subset S of a vector space, S is the generating set of $L(S)$, linear span of a non empty subset of a vector space is a subspace of the vector space, Linearly independent / Linearly dependent sets in a vector space, properties such as; a set of vectors in a vector space is linearly dependent if and only if one of the vectors v_i is a linear combination of the other vectors $v_j' s$.

Unit III: Basis and Linear Transformation (15 Lectures)
Basis of a vector space, Dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any set of $n+1$ vectors in a vector space with n elements in its basis is linearly dependent, any two basis of a vector space have the same number of elements, any n linearly independent vectors in an n dimensional vector space is a basis of a vector space, if W_1 and W_2 are subspaces of a vector space then $W_1 + W_2$ is a subspace of the vector space, $\dim(W_1 + W_2) = \dim W_1 \quad + \dim W_2 - \dim(W_1 \cap W_2$.), extending the basis of a subspace to a basis of a vector space.

Linear transformation, kernel, matrix associated with a linear transformation, properties such as; kernel of a linear transformation is a subspace of the domain space, for a linear transformation T, $image\ (T)$ is a subspace of the co-domain space, if V, W are vector spaces with $\{v_1, v_2, \dots v_n\}$ a basis of V and $\{w_1, w_2, \dots w_n\}$ any vectors in W then there exists a unique linear transformation $T: V \to W$ such that $T(v_i) = w_i, 1 \leq i \leq n$, Rank nullity theorem (only statement) and examples.

Recommended Books
1. Serge Lang, Introduction to Linear Algebra, Second Edition, Springer.
2. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India, Pvt. Ltd, 2000.

Reference Books
1. M. Artin: Algebra, Prentice Hall of India Private Limited, 1991.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi, 1971.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
3. L. Smith: Linear Algebra, Springer Verlag.
4. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
5. T. Banchoff and J. Warmers: Linear Algebra through Geometry, Springer Verlag, New York, 1984.
6. Sheldon Axler: Linear Algebra done right, Springer Verlag, New York.
7. Klaus Janich: Linear Algebra.
8. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
9. Gareth Williams: Linear Algebra with Applications.

Assignments (Tutorials)
1. Solving homogeneous system of m equations in n unknowns by elimination for $(m,n) = (1,2), (1,3), (2,2), (2,3),\ (3,3)$, Row echelon form.
2. Solving system $AX = B$ by Gauss elimination, Solutions of system of linear Equations.

3. Verifying whether $(V, +, .)$ is a vector space, for a given set V and w. r. t. $'+'$ and $"."$.
4. Linear span of a nonempty subset of a vector space, determining whether a given subset of a vector space is a subspace. Showing the set of convergent real sequences is a subspace of the space of real sequences etc.
5. Finding basis of a vector space such as $P_3[X], M_2\ [R]$ etc. Verifying whether a set is a basis of a vector space. Extending basis to a basis of a finite dimensional vector space.
6. Verifying whether $T: V \to W$ is a linear transformation, finding kernel of a linear transformations and matrix associated with a linear transformation, verifying the Rank Nullity theorem.

Scheme of Examination for Semester I&II

The performance of the learners shall be evaluated into two parts. The learner's performance shall be assessed by Internal Assessment with 40% marks in the first part by conducting the Semester End Examinations with 60% marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

(a) Internal assessment 40 %

Courses with tutorials (Mathematics)

Sr No	Evaluation type	Marks
1	One Assignments (one Tutorial converted into assignment) / Case studies / Project	10
2	One class Test [Tutorial converted into test]	20
3	Active participation in routine class instructional deliveries/Tutorials	05
4	Overall conduct as a responsible student, mannerism and articulation and exhibit of leadership qualities in organizing related academic actives	05

(b) External Theory examination 60 %

1. Duration – Theses examinations shall be of 2 Hours duration.
2. Theory Question Paper Pattern:-There shall be four questions each of 15 marks. On each unit there will be one question and the fourth one will be based on entire syllabus. All questions shall be compulsory with internal choice within the questions. (Each question will be of 20 to 23 marks with options.) Question may be subdivided into sub-questions a, b, c… and the allocation of marks depend on the weightage of the topic.

Guidelines about conduct of Tutorials/Assignments

1. Tutorials

Conduct and Evaluation: The tutorials should be conducted in batches formed as per the University circular. The tutorial session should consist of discussion between the teacher and the students in which students should participate actively. Each tutorial session should be evaluated out of 10 marks on basis of participation of student and the average of total aggregate should be taken.

2. Assignments

Conduct and Evaluation: The topic of the assignment and the questions should be given to the students at least one week in advance. The assignment should be such that it can be completed in 45 - 50 minutes by a student. The teachers may resolve the doubts of the students during the week, after which the students should submit the assignment. Each assignment should be evaluated out of 10 marks and the average of the total aggregate should be taken.

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