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DOES CHAOS IMPLY CONTROL ? A REACTION TO THE LUCAS CRITIQUE

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DOES CHAOS IMPLY CONTROL?: A REACTION TO THE LUCAS CRITIQUE

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The paper shows that a control model incorporating a simple policy rule can yield chaotic trajectories for the state variables implying that a random apparently nonsystematic policy can still be considered as policy. Such a policy rule is then applied within the framework of a monetary surprise model in order to show that it in- possible, under certain parametric restrictions, to raise the growth rate of output beyond its natural rate.

1. Introduction

The paper by Lucas (i976) on econometric policy evaluation is widely considered as the most formidable challenge ever made to the legitimacy of econometrics applied to macroeconomics and to the basis of control theory applied for policy analysis.

The Lucas critique is merely the observation that if a particular policy regime is constant the estimated parameters of an econometric model will be constant, but if the regime changes the estimates of the parameters will also change, implying that an econometric model is useless for evaluating alternative policy rules because, having been estimated under one regime, they will necessarily mispredict under a different regime. Only if the effect of a regime change on the underlying model can be adequately specified would an empirically estimated model be of any use. However, this presupposes that the underlying model is the true model which could be a very tenuous assumption to make.

One of the several responses to Lucas' argument states that while the Lucas critique is correct in principle, true changes in policy regime are rare, so that even models which would not he invariant across regime changes will still be useful for forecasting and policy analysis within a given regime. Such a statement is paradoxical to say the least because it argues that when there is no change of regime, it is possible to evaluate policy without being pre-empted by the Lucas critique. But if policy makers act on such advice it would imply a change in regime, thereby invalidating the exercise. This paradox revolves around the problem of how policies should be characterized which is the problem we intend to examine in this paper.

2. <u>The Characterization of Policy</u>

A policy regime is identified with a rule governing the evolution of a policy variable. For example, assume that the central bank supplies money according to the rule:

$$m_t = \mu m_{t-1} + e_t \tag{2.1}$$

where mt is the natural logarithm of money supply at time t, and et is a random error with mean zero. In this formulation, each possible value of y characterizes a different policy regime. To evaluate a change of policy regime within a model, the model is usually solved with alternative values of μ. Since rational expectations usually involves infinite time horizons, this procedure implicitly assumes that once policy is changed it will be maintained at that regime forever. Several researchers have argued that this characterization of a change is flawed on two counts: first, it assumes a strict form of rational expectations and, second, it assumes that policies never change at all.

In this paper, we shall examine only the second of these objections: that policies do in fact change and are known to change. Consequently, rational expectations would assign probabilities to the current policy continuing in force as well as to each possible policy succeeding it at some future time.

Cooley, LeRoy and Rayrnon (1984a, b) argue strongly that policy should always be treated as variables and not as fixed parameters. Under this viewpoint, the money supply rule, eq. (2.1), might still be considered as describing policy, but y would now be regarded as time invariant and a change in polioy regime would be represented as different sequences of et.

Although, this view appears to be an attractive alternative to the difficulties present in Lucas[1] parametric characterization of policy, it has its own problems. The public views et as a aerially uncorrelated random variable. In that sense, it in not possible to assume that the authorities can choose it. One way out suggested by Cooley et al. would be to say that et is random from the viewpoint of the public although it is chosen by th« authorities, i.e., the public views the authorities as a black box which generates values for m^ by an unknown process.

However, such a view has not found much support from new classical analysts because they feel that it is not compatible with rational expectations. For if the public has rational expectations then its subjective beliefs about et correspond to what is objectively true. If they believe that et follows a particular random procees then the authorities can choose e-only in a manner that appears to follow that process and such a constrained choice is no choice at all (see Hoover 1988, p. 1995) .

On the other hand, allow et to be an uncontrolled random error and permit the authorities to choose v, say, v1 or $V2^m$ Cooley et al then proceed to argue that as p is now a variable, the public with rational expectations would know the probabilities of a switch between y1 and V2 so that both would belong to an identical regime. But, once again, the same question arises: if subjective beliefs about the probabilities of switching between u1 and u2 are objectively true, then the authorities have no real choice about y. It is possible, in this context, to assume that the authorities choose not y^ and ^2 but rather the probabilities of switching which, of course, need not be fixed so that a regime change would be a change in the probabilities of switching. But then a rational public would know the probabilities of switching between y^ and H2* The problem can ramify to ever higher levels but regardless of the multi-stage decision making process involved, the policy maker has no real choice if the public's estimates of the transitional probabilities are objectively true.

However, according to Cooley et a], the process cannot be ramified forever. Their view is that the public situates 'new' events as repetitions of previous patterns,- but the public's expectations can be rational only if the authorities in fact conform to previous patterns, complicated though they may be. At some level, argue Cooley et aj, the authorities can surely choose probabilities or parameters for which there is absolutely no precedent. The public would, under the circumstances, be unable to form objectively correct assessments of the probabilities. This is now a change of regime.

One of the main reasons as to why the arguments of Cooley eh al have not found much favour in the literature is that the source of randomness in Cooley et al'a (1984b) model illustrating their view of policy as generated by a black box has not been explained (see Sims 1986, p. 299). Despite this failing, it will be shown in this paper that the basic logic of their argument is correct as it seems to provide the only feasible framework by which the Lucas critique can be pre-empted.

To summarize the issues needed to be resolved to circumvent the Lucas critique: (i) the authorities should possess a policy rule which generates values for the policy variables by a known systematic process, (ii) the public should view the resulting policy as being randomly generated, and (iii) the policies generated should be such that new events should neither have a precedent nor bear any resemblance to previous patterns.

This would then imply that the public, although it has rational expectations, would not be able to correctly form objective assessments of the probabilities inspite of possessing subjective beliefs regarding the apparent randomnesn of policy. Moreover, there can be no objections that what is being implemented is not policy because although policy might look nonsystematic, it is in fact being generated using a systematic rule which is capable of allowing the policy maker to affect the probabilities assigned to different realizations of the resulting distributions. The model in this paper will confirm that, by tins moving the problem up one level from Lucas original fixed and random-parameter illustration Cooley et al had in fact anticipated a possible solution to the Lucas critique.

3. control Theory And Feedback Rules

While it is true that Sims (1980, 1982) had anticipated Cooley et al.'s argument that policy should be described by a variable with a well-defined probability distribution, he noted later on that the distinction between choosing a fixed parameter in a policy rule and choosing a sequence of policy variables was superficial (Sims 1986a, p. 297), because even a permanently fixed parameter could pose difficulties for the application of rational expectations during the transition between regimes.

In this, however, Sims was echoing the sentiments of many others including Taylor (1975) and Bray (.1.982, 1983) who believed that a strict form of rational expectations, i.e., one where people understood the new policy and incorporated its effects into their decisions .instantly, was untenable. On the other hand, a weaker form of rational expectations, i.e., one where people learned from their past mistakes, was considered more plausible.

Lucas (1977, 1986) accepted the critical aspect of this argument: if such a weaker form holds, it is important to model the learning process because the characteristics of an economy in transition between regimes will be quite different from its characteristics after the policy has been in force for some time.

Where Sims differed from Lucas was that he believed private individuals have seen a sufficiently wide range of government actions and are able to attach probabilities to these actions recurring, so that true regime changes are really very rare (Sims 1982). By viewing the public and the authorities an optimizing agents, Sims (1986a) argued that game theory, rather than control theory, should be the basis of policy analysis.

Sims view of policy making from an econometric perspective is similar to that of Barro and Gordon (1983a, b) as both of "them raise the same theoretical issues: if policy makers and the public are both behaving optimally, then there is no possible, role left for giving advice on policy.

Moreover, Sims also believes that even if policy makers follow complex feedback rules which practise 'leaning against the wind', just the mere fact that the authorities have followed some deterministic feedback rule in the past does not provide the econometrician with the ability to obtain the requisite information to give useful advice.

While Sargent (1984) believes that Sims has pushed too far the hypothesis that agents optimize, the argument posed by the presence of feedback rules in a control theoretic framework seem to undermine Sargent's claim. To see this, consider the following control equation where output at time t, yt, is a function of: pant output, yt-l and a current policy variable, xj, i.e.

$$y_t = \alpha y_{t-1} + \beta x_t + \epsilon_t \tag{3.1}$$

where ϵ_t is a random error. Assume that the authorities have; followed the following deterministic feedback rule of the form:

$$x_t = \phi y_{t-1}$$

It might seem obvious that an econometrician could be able to estimate the effect of such a policy rule on output in ordfer to help the authorities choose an optimal value for if *however* the authorities have been following any feedback rule in the past, this is not possible. To prove this, substitute eg (3.2) into eq. (3.1) to yield:

$$y_t = (\alpha + \beta\phi)y_{t-1} + \epsilon_t = \pi y_{t-1} + \epsilon_t \tag{3.2}$$

Which indicates that, because of the deterministic nature of the feedback rule, the control variable xt vanishes from the reduced form. Even if one knew the exact value of the parameters a and 0 cannot be recovered from the estimate of w: they are not identified. Thus, although x^ is used to control yt, xt does not Granger-cause yt thereby illustrating that Granger-causality and controllability may in fact be opposed to each other.

While recognizing that the above argument is correct, given its assumptions, Sims (1986a, p. 298) believes that it is wrongly formulated because policy rules in the real world are not deterministic but subject to, what he refers as, capricious variations. Thus, Sims'a suggests replacing eq (3.2) by:

$$x_t = \phi y_{t-1} + \sigma_t \quad (3.4)$$

where o^ is a random error representing the uncertain element of policy implementation. Substituting eq. (3.4) into eq. (3.1):

$$y_t = (\alpha + \beta\phi)y_{t-1} + \beta\sigma_t + \epsilon_t \quad (3.5)$$

Using eq. (3.4) in order to eliminate ot yields:

$$y_t = (\alpha + \beta\phi)y_{t-1} + \beta x_t - \beta\phi y_{t-1} + \epsilon_t$$

$$= \alpha y_{t-1} + \beta x_t + \epsilon_t \quad (3.6)$$

Which is identical with eq. (3.1), thereby demonstrating in the presence of capricious variations, the econometrician may once more be able to estimate controlled behavioural relations.

However, as Hoover (1988, p. 201) has mentioned, "...if tho capricious element results not from a failure of individual groups to pursue their own goals optimally but from the interaction between different groups with different goals, it- remains difficult to see what use any group would find for information obtained econometrically".

if a group already follows an optimal policy, it has no fixed to evaluate alternative policies and therefore the Lucas critique in irrelevant. Thus, Sims U9H0, p. 14) concludes that the evaluation of policy is a moot issue. The authorities know precisely what they wish to do, although they do not know exactly how to implement such a policy. Thus, Sims reserves only a modest role for the eeonometrician: that of policy implementation.

To elucidate this, consider the earlier example. Suppose that the desired policy is to minimize the variability of output. The best that the authorities can do under the circumstances in to set xt such that the deterministic component of the right-hand side of eq. (3.1) is zero, i.e.,

$$\alpha y_{t-1} + \beta x_t = 0 \tag{3.7}$$

which implies that:

$$x_t = -(\alpha/\beta) y_{t-1}$$

or, equivalently, to set:

$$\phi = -\alpha/\beta \tag{3.9}$$

The only way to implement this is, of course, to estimate a and β, and then to calculate ϕ. However, as we have shown, even in the present simple example, this is not straightforward because of the identification problem: it would be impossible to determine the analogues to a and β in order to choose

In order to resolve this, we show here that it becomes unnecessary to distinguish between policy evaluation and policy implementation provided that the feedback rule of the control system is so formulated that it possesses the advantage of yielding the required structural information on the mode based upon which an entire range of policy options can be examined.

We then prove that the control model, by yielding chaotic trajectories, overcome the failing of Cooley at al by specifying precisely the source of randomness which would substantiate the view that the public would have to consider policy an being randomly generated by a black box with new events neither having a precedent nor bearing any resemblance to previous patterns. Under the circumstances, despite the fact that the public is also an optimizing agent, it would be unable to objectively form correct assessments of the probabilities and therefore it would have no other option other than being passively manipulated.

The advantage of such a procedure would be that in such a situation, the weaker form of rational expectations would not be valid because as each policy regime is a completely new one, people would never be able to learn from their past mistakes. As the domain of such a theoretical framework would be a continuously recurrent phenomena with no apparent steady state, there would be no possible paths between one steady state to another and therefore it would be impossible to model the learning process. Thus, the observation by Lucas (1977, p.15) regarding the unsubstantive nature of the weak form of rational expectations would be true. At the same time, however,his contention that economics is about the common steady states and has nothing to say about these transitional paths would be invalid because the economy would never be able to settle down close to its steady state path as people, being unable to understand the characteristics of any policy, would never be able to adjust to a change in regime, thereby pre-empting even the strong form of rational expectations.

4. From Control To Chaos

4.1 A model of chaos

This section provides a brief discussion on the chaos model that we shall be invoking in order to ultimately substantiate our proposition. Chaos is a process in which a dynamic mechanism that could be very simple and, above all, deterministic yields a time path so complicated that it will invariably pass all the standard tests of randomness which are unable to distinguish such chaotic patterns from truly random behaviour. Moreover, truly chaotic trajectories never ever return to any point previously traversed, but display over a bounded region, often referred to as an attractor, a "disorderly" oscillatory pattern.

In all chaotic models, the system follows nonlinear dynamics which are self-generating and never die down and it is the fact that fluctuations can be internally generated, without; recourse to exogenous shocks, that has proved to be intuitively appealing.

Brock (1986) provides the exact mathematical definitions and formulations of chaos while Baumol and Benhabib (1989) survey economic models which produce chaotic behaviour. While many complex models of chaos have been specified in the literature, e.g. the Henon map, the Lorenz model the van der Pol system, the Rossler band, the Mackey-Glass equation, amongst others, the simplest and most common chaos model employed in mathematics or in economics is the logistic curve which involves a univariate nonlinear first-order difference equation whose functional representation is of the form:

$$x_{t+1} = Ax_t(1-x_t) \qquad (4.1)$$

where $0 < x_0 < 1$ and $0 < A < 4$.

It has been shown that for small values of A, the system in, stable, but as the value of A approaches 4, the system becomes chaotic. In essence, the logistic model sums up the four basic properties of chaotic behavior: One, |xt|J fills up the unit interval 10,1J uniformly as t -> ∞. Technically, this implies t-hat the fraction of points in (xt} falling into an interval (a,b) is (b-a) for any 0 < a < b < I. Two, any small error in measuring the initial XQ will be compounded exponentially fast in forecasts of xt. This property is usually referred to as "sensitive dependence on initial conditions". Three, Xt appears stochastic even though it is generated by an entirely deterministic process, in the sense that the empirical auto covariance function:

$$\phi_{xx}(k) = E[x_t x_{t-k}] = \lim_{T \to \infty} \left[\sum_{t=0}^{\infty} x_t x_{t-k} / T \right] = 0 \quad \ldots(4.2)$$

which is the same as that of white noise. Four, the dynamics of a chaotic system depend on parameter values.

While the above analysis has been based on the analytical behaviour of a particular family of functions $\{f_r\}$ whose phase graph is hill-shaped, one of the more surprising conclusions produced by mathematical theory is that similar conclusions can be obtained for a wide class of families of functions (gr) provided they are single-peaked, possess a negative Schwarzian derivative, and are. increasing in a given parameter.

Thus, the results produced by chaos theory are independent of the particular family of functions |Fr|and, based upon this fact, we attempt to provide a framework which would enable us to convert a control model into a chaos model of the logistic type

4.2 A model of control

While much work has been carried out in recent yearn addressing the problem of macroeconomic control in the presence of uncertainty, the examination of the sensitivity of the implemented control to error between the estimated model and the true system has been unfortunately neglected (see Rao 1990).

State variable feedback (SVF) with constant or time-varying gains implies only 'proportional plus derivative* control, and unless applied in problems formulated in stochastic terms with certain non-stationary disturbances, e.g. in Kalman filtering methods, the basic SVF control approach does not yield a control law incorporating direct 'integral action[1], i.e., the feedbaok control law comprises no error terms which are related explicitly to the integral of error between the desired target levels and the actual attained levels of the state variables.

While control theorists have argued that the integral of error between the desired target levels and the model variables is implicit in the tracking gains of the feedback control la*, given by eq. (3.2), such an impersonation is of little consequence in the presence of model misspecification since t»e minimization of the model-target error will almost definitely presuppose the existence of error or 'offset' between the actual outcomes and their specified targets if the estimated model s not: an exact representation of the actua1 system.

There are a number of methods available for overcoming the Problem of offset in SVF control systems. Perhaps the simple-it: approach is the multivariable equivalent of the single variable Proportional-Integra1-Derivative (PID) control method.

Consider the state transition equation of the following linear control model:

$$x_{t+1} = Ax_t + Bu_{t+1} + Cz_{t+1} \qquad (4.3)$$

where xt denotes an n-vector of state variables at time t_f u^ denotes an m-vector of control variables, zt denotes a p-vector of exogenous (uncontrollable) variables; and A, B, and C are appropriately dimensioned matrices.

In a PID control system, an integral of error action \n introduced through the augmentation of the state space to include a vector of additional state variables e_{t+1} defined as the integral of error between the state vector x_{t+1} and the target levels for this vector denoted by x (which can be time-varying if so desired). Thus, the 'integral error[1] vector is defined by;

$$e_{t+1} = e_t + [x_{t+1} - \tilde{x}] \qquad (4.4)$$

However, in our model, for the purpose of converting the PID control system into a chaotic process, we replace the integral error vector by a quadratic error vector given by:

$$e_{t+1} = (x_{t+1} - \bar{x})^2 \qquad (4.5)$$

which can be considered as a squared loss function.

We now assume a feedback control law of the form:

$$u_{t+1} = G_1 x_t + G_2 e_t \qquad (4.6)$$

where G1 is the 'proportional cum derivative' feedback gain of the system which is analogous to in eq. (3.2), and G2 iS the 'quadratic' feedback gain of the system which now plays the role of the 'integral' feedback gain. In the conventional literature , the role of these two feedback gains, Gj and G2, is to relocate the eigen values of the resulting closed-loop system at any arbitrarily specified locations in the complex plane.

In this paper, as we will be discussing the most general 'version of the Lucas critique, i.e., the effect of money supply (a single control variable) on output (a single state variable) we shall consider only the single variable equivalent of the generalized multivariable Proportional-Integral-Derivative (PfD) control method. Under the circumstances, the role of these two feedback gains will be quite different: in effect they will be used to ensure that? (i) the resulting single-variable system is a chaotic process, and (ii) the desired equilibrium values of such a chaotic system are stable.

Before we set about doing so, we will initially have to prove that, for such a single-variable control system, an econometrician would be able to estimate the effect of money supply on output in order to help the authorities choose desirable values for G1 and G2* To see that this is indeed the case, substitute eq. (4.5) into eq. (4.6), and the resulting expression into eq. (4.4) to yield:

$$x_{t+1} = (BG_2\bar{x}^2 + Cz_{t+1}) + (A + BG_1 - 2BG_2\bar{x})x_t + (BG_2)x_t^2$$
$$= \pi_0 + \pi_1 x_t + \pi_2 x_t^2 \qquad (4.7)$$

Now eq. (4.7) tells us that although, because of the deterministic nature of the PID feedback rule, the control vector vanishes from the estimated reduced form representation, it would be possible, if one knew the exact values of Gj and G2 and $\bar{x}$, to recover the parameters A, B, and C from the estimates of π_0, π_0, and π_2 thereby implying that they are identified. Consequently, it is unnecessary to introduce 'capricious variations' in the realizations of policy rules a l_a Sims in order to enable the econometrician to estimate controlled behavioural relations.

Thus, the criticism of Sims is no longer valid in the above control scheme because even if policy rule are completely deterministic and if the authorities have been following this feedback rule in the pant, it would still be possible for an eoonometrician to identify an optimal policy rule based oh the knowledge of the parameters of the behavioural relations given by eg. (4.3). We now intend to obtain the requisite information based upon which it would be possible to evaluate policy.

In order to do so, we rewrite eq. (4.7) as follows

$$x_{t+1} = \pi_0 + \pi_1 x_t [1 + (\pi_2/\pi_1)x_t] \qquad (4.B)$$

which, if $\pi_1 > 0$ and $(\pi_2/\pi_1) < 1$, is immediately seen to bear a resemblance to our logistic-curve chaos model given by eq. (4.1). Thus, for our control model to generate chaotic trajectories for Xt, the following reduced-form restrictions must hold:

$$\pi_1 = A + BG_1 - 2BG_2\bar{x} > 0 \qquad (4.9a)$$

$$\pi_2/\pi_1 = BG_2/(A + BG_1 - 2BG_2\bar{x}) < 0 \qquad (4.9b)$$

If, in our single-variable case, we assume that A, B, and x are positive, eqs. (4.9a) and (4.9b) together would imply that:

$$G_1 > (2BG_2\bar{x} - A)/B \qquad (4.10A)$$

$$G_2 < 0 \qquad (4.10b)$$

The phase curve is defined as the graph of eq. (4.8), for given fixed values of π_0, π_1, and π_2, and expresses x_{t+1} as a function of X(-. The slope of this phase curve is given by:

$$\delta x_{t+1}/\delta x_t = \pi_1 + 2\pi_2 x_t \qquad (4.1\ )$$

We see from eq. (4.8) that whatever be the values of π_0, π , and π_2, the phase curve must always reach its maximum at:

$$x_t = -\pi_1/2\pi_2 \qquad (4.12)$$

at which point its slope, given by eq. (4.11), would be zero.

At that point, its maximum height (x_{max}) would be equal to:

$$x_{max} = \pi_0 - (\pi_1^2)/4\pi_2$$

which in seen to increase with π_0 and π_2, and decrease with .We also realize that the equilibrium value of x in linked to the three parameter**, π_0, π_1, and π_2, of the difference equation because, at equilibrium, we must have: $x_{t+1} = x_t = x^*$, HO that:

$$x^* = \pi_0 + \pi_1 x^* [1 + (\pi_2/\pi_1)x^*] \tag{4.14}$$

which is a quadratic in x*. Solving it yields the following two equilibrium solution of the chaos model given by:

$$x^*_1 = [-(\pi_1-1) + \{(\pi_1-1)^2 - 4\pi_2\pi_0\}^{\frac{1}{2}}]/2\pi_2 \tag{4.15a}$$

and

$$x^*_2 = [-(\pi_1-1) - \{(\pi_1-1)^2 - 4\pi_2\pi_0\}^{\frac{1}{2}}]/2\pi_2 \tag{4.15b}$$

Moreover, from eqs. (4.11) and (4.15), we note that the slopes of the phase graph at these two equilibria are given by:

$$\pi_1 + 2\pi_2 x^*_1 = 1 + \epsilon \tag{4.16a}$$

and

$$\pi_1 + 2\pi_2 x^*_2 = 1 - \epsilon \tag{4.16b}$$

where $\epsilon = \{(\pi_1-1)^2 - 4\pi_2\pi_0\}^{\frac{1}{2}}$.

Based upon the theory of difference equations, according to which an equilibrium solution will be stable if and only if th« absolute value of the slope of the phase graph at equilibrium *is* strictly less than unity, the above analysis provides the basic for five general cases: (1) If $-2 \geq \epsilon$, then both the equilibrium Solutions will be unstable; (2) Tf $-2 < \epsilon \leq 0$, then x^*_1 will be Stable but x^*_2 will be unstable; O) If 6 = 0, then both the equilibrium solutions will be unstable; (4) If $0 < \epsilon < 2$, then x^*_1 will be unstable but x*2 will be stable; and (5) if $\epsilon \geq 2$, then both the equilibrium solutions will be unstable.

As € depends upon the reduced-form parameters of the chaos - model, π_0, π_1, and π_2; which in turn depend upon the structural-' form coefficients of the control model, A, B, and C (which have been shown to be recoverable from the reduced-form estimates) as well as the feedback coefficients of the PID control law, Gj and G2; it is obvious that by choosing the correct values for G1 and G2 it would be possible to either select (i) the desired equilibrium value towards which the system should converge, or (ii) the desired pair of equilibrium values around which the system should oscillate chaotically.

Given the standard results in chaos theory (see Li and Yorke 1975), there will be some set of values for the controlling parameters of the system, π_0, π_1, and π_2, for which three-period cycles will theoretically occur. At this juncture, there will be an uncountable number of initial values yielding bounded time paths which never repeat past behaviour, no matter over how long a set of time periods the iterations are continued. Thus, while there is an explicit analytic formula for the solution of the system, to any observer, who is unaware about the underlying structure of the system, the resulting trajectory will be indistinguishable from one generated by a totally random proceas.

This is precisely the source of randomness that was first alluded to by Cooley et al while illustrating their view on policy and we have now formalized the source of such randomness. Using the above results, we will show, within the framework of the 'monetary surprise[1] model, that the Lucas critique, even assuming a strict form of rational expectations, would be invalid in the presence of such an explicit source of randomness.

5. Money And Output: An Analytical Framework

Our objective here in to set forth a particular model to be used in the subsequent macroeoonomic analysis of evaluating the Lucas critique. Ideally, it should be possible to convert such a. model into a control theoretic framework so that its chaotio analogue can easily be derived on the lines suggested in the lash section. In an attempt to develop such a model, we will assume a relatively streamlined structure which takes into account only those aspects which are necessary for the purpose of the paper*

5.1 Aggregate Demand; Basic Model

In implementing this strategy, it will be convenient to have a specific representation which models the economy's aggregate demand relations. Accordingly, we assume that the saving-investment relationship and money demand behaviour can be represented within an IS-LM framework.

Let the money demand function be given by:

$$m_t - p_t = a_0 + a_1 y_t + a_2 r_t + \phi_t \qquad (5.1)$$

where mt is the level of nominal money stock in period t, pj. in the actual level of prices in period t, yt is the level of real output in period t, rt is the nominal interest rate in period t, and at is a serially uncorrelated random error with mean of zero. (Throughout this section, lower case letters indicate the natural logarithm of variables). Here the parameter signs are: $a1 > 0$, and $a2 < 0$. This LM function summarizes behaviour pertaining to money demand and supply. We also assume that the money stock mt. is directly manipulated by the monetary authorities.

Next for the IS relationship summarizing saving and investment behaviour, we assume that:

$$r_t = b_0 + b_1 y_t + \theta_t \quad (5.2)$$

where 0t is a random error term with mean zero to reflect shocks to saving and investment. Here the parameter sign is b1 < 0 reflecting the negative relationship between output demand and the rate of interest.

Substituting eq. (5.2) into (5.1), it is possible to derive an aggregate demand function given by:

$$m_t - p_t = a_0 + a_1 y_t + a_2 [b_0 + b_1 y_t + \theta_t] + \phi_t \quad (5.3)$$

Solving" the above equation for y^ yields:

$$y_t = \beta_0 + \beta_1 (m_t - p_t) + \epsilon_t \quad (5.4)$$

where: $\beta_0 = -(a_0 + a_2 b_0)/(a_1 + a_2 b_1)$ and $\beta_1 = 1/(a_1 + a_2 b_1)$. Given the parametric restrictions, we have: $\beta_1 > 0$. Also, Et is a composite stochastic disturbance: $\epsilon_t = -(\phi_t + a_2 \theta_t)/(a_1 + a_2 b_1)$.

Eq. (5.4) indicates that the quantity of output demanded in period t is positively related to the current magnitude of real money balances. In the formulation above, we have included no fiscal variables, but that omission has been made only for simplicity because, in general, government spending would show up in an IS function such as eq. (5.2).

5.2 Aggregate Supply: Basic Model

In order to develop a model of aggregate supply, the general approach is to presume that product prices are set at the start, of each period at expected market-clearing levels. However, these prices do not change within the period, so that unexpected shock* to demand or supply will cause the actual market-clearing valued to diverge away from their expected values. In such cases, the quantity produced and sold will be that specified at the prevailing price by the demand function.

The idea underlying the above assumption is that .suppliers find it advantageous to preset their prices and then satisfy whatever demands emerge at those prices, even if they exceed the quantities they would normally wish to supply. Thus, at the aggregate level, the quantity of output that is actually exchanged in any period will be determined, once the price **level** Pt is given, by the aggregate demand function eq. (5.4).

The manner in which "supply-side" considerations enter the framework is via the determination of p£ and, as stated earlier, the principle governing this determination is that pt is preset at the expected value of the market-clearing price which is denoted by pt. Thus, we have:

$$p_t = E_{t-1}\bar{p}_t \tag{5.5}$$

where the use of the conditional expectation operator Et-1 indicates that rational expectations is being assumed in the analysis and that these expectations are based on all information available uptil time $t-1$, i.e., Ω_{t-1}.

For eq. (5.5) to be meaningful, we have to define the na turn of pt. In order to do so, let us define y, which is the (assumed) constant natural rate of output determined by technology and resources, as that value of yt which corresponds to a market-clearing situation in the labour market.

Now, by definition, the market-clearing price pt if the value of pt that would equate aggregate output demanded, given by eq. (5.4), with y. Therefore, in the present framework, we have:

$$\bar{y} = \beta_0 + \beta_1(m_t - \bar{p}_t) + \epsilon_t \tag{5.6}$$

Taking the mathematical expectation of both sides given all the information available at time t-1, i.e., given Ω_{t-1}, yields:

$$E_{t-1}y = \beta_0 + \beta_1 E_{t-1}(m_t - p_t) + E_{t-1}(\epsilon_t) \tag{5.7}$$

Now as the natural rate is assumed to be a constant, its Value will be known in advance, so that $E_{t-1}\bar{y} = \bar{y}$. Also, from eq. (5.5), we have pt= $E_{t-1}\bar{p}_t$. Since, Et is assumed to be random, its bent predicted value is its mean E= 0 . Therefore, we have

$$\ddot{y} = \beta_0 + \beta_1(E_{t-1}m_t - p_t) \tag{5.8}$$

Subtracting eq. (5.8) from eq. (5.4) yields:

$$y_t - \bar{y} = \beta_1(m_t - E_{t-1}m_t) + \epsilon_t \tag{5.9}$$

which can be rewritten as:

$$y_t = \bar{y} + \beta_1(m_t - E_{t-1}m_t) + \epsilon_t \tag{5.10}$$

which is our desired solution for yt as it expresses the manner in which output responds to variables (policy and expectational) and shocks (exogenous).

In particular, it is seen that output deviates from its market-clearing value y whenever a nonzero shock affects aggregate demand (i.e., $\epsilon_t \neq 0$)›netary policy shock occurs (i.e., $m_t \neq E_{t-1}m_t$). Furthermore, since the parameter $i is positive, a positive monetary surprise (i.e., $m_t > E_{t-1}m_t$) will give rise to an output level that is higher than y and, in that sense, higher than normal.

Based upon several important results, researchers who have examined the Lucas critique within the framework of the 'monetary surprise[1] model have unequivocally concluded that, regardless of the policy* or feedback rule that is adopted, there is no systematic monetary policy that will keep yt p, manently higher or lower in relation to y.

We will now re-examine this conclusion by applying the results on control and chaos theory obtained in Section 4.

6. Evaluating the Lucas Critique

6.1 A Monetary-Surprise Model of Chaos

To convert the monetary-surprise model into a chaotic process, we initially re-write eq. (5.10) as a control model, i.e.

$$y_t = \beta_1 m_t + (\bar{y} - \beta_1 E_{t-1} m_t) \quad (6.1)$$

where, by assuming certainty equivalence, we have net 6^ equal to its mean of zero. In the above single-variable control formulation, yt is the state variable, m^ is the control variable and y and $E_{t-1}m_t$ are the two exogenous variables. Comparing eq. (6.1) which is seen to be analogous to eq. (4.3) vields? the following correspondence: A=O ; B = P1; and Czt = $(\bar{y} + E_{t-1}m_t)$.

In our context, systematic monetary policy is described by the initial choice of an appropriate squared loss function and then by the subsequent choice of a PID feedback control law. Let our quadratic error term be given by:

$$e_t = (y_t - \bar{y})^2 \quad (6.2)$$

which implies that the monetary authorities are interested in ensuring that the actual level of output, y_t, tracks the constant natural rate of output, y.

Let our feedback control law be of the form:

$$m_t = G_1 y_{t-1} + G_2 e_{t-1}$$

which implies that the monetary authorities react to previous levels of output as well as to previous values of the squared loss function and set money supply accordingly.

In such a context, the question of whether or not monutary policy can systematically affect output is the question of whether or not the policy parameters G] and G2 are part of the solution of the model for output yt.

In order to answer this question, we substitute eq.(6.2) into eq.

$$m_t = G_1 y_{t-1} + G_2(y_{t-1} - \bar{y})^2$$
$$= G_1 y_{t-1} + G_2 y_{t-1}^2 - 2G_2 \bar{y} y_{t-1} + G_2 \bar{y}^2 \quad (6.4)$$

We now substitute eq. (6.4) into eq. (6.1) to yield:

$$y_t = \beta_1(G_1 y_{t-1} + G_2 y_{t-1}^2 - 2G_2 \bar{y} y_{t-1} + G_2 \bar{y}^2) + (\bar{y} - \beta_1 E_{t-1} m_t)$$
$$= (\bar{y} + \beta_1 G_2 \bar{y}^2 - \beta_1 E_{t-1} m_t) + (\beta_1 G_1 - 2\beta_1 G_2 \bar{y}) y_{t-1}$$
$$+ (\beta_1 G_2) y_{t-1}^2 \quad \ldots(6.5)$$

Eq. (6.5) can thus be. written in the form

$$y_t = \pi_0 + \pi_1 y_{t-1} + \pi_2 y_{t-1}^2 \quad (6.6)$$

where:

$$\pi_0 = \bar{y} + \beta_1 G_2 \bar{y}^2 - \beta_1 E_{t-1} m_t$$
$$\pi_1 = \beta_1 G_1 - 2\beta_1 G_2 \bar{y}$$
$$\pi_2 = \beta_1 G_2 \quad (6.7c)$$

It is now obvious that eq. (6.6) has the same form an the Logistic model of chaos given by eq. (4.8) provided that the Following parametric restrictions hold:

$$\pi_1 = \beta_1 G_1 - 2\beta_1 G_2 \bar{y} > 0 \quad (6.8a)$$
$$\pi_2 = \beta_1 G_2 < 0 \quad (6.8b)$$

which, under the assumption that $\beta_1 > 0$, implies that:

$$G_1 > 2G_2 \bar{y} \quad (6.9a)$$
$$G_2 < 0 \quad (6.9b)$$

The slope of the phase graph for this chaotic monetary surprise' model is obtained by differentiating by with respect to *Yt-1* in eq. (6.6). We thus obtain:

$$\delta y_t / \delta y_{t-1} = \pi_1 + 2\pi_2 y_{t-1} \quad (6.1)$$

Which,using eqs. (6.7b) and (6.7c), yields:

$$\delta y_t / \delta y_{t-1} = \beta_1 G_1 - 2\beta_1 G_2 \bar{y} + 2\beta_1 G_2 y_{t-1} \quad (6.11)$$

6.2 Equilibrium in the monetary-surprise model

In such a system, the equilibrium value of yt is obtained by initially setting yt • $y_{t-1} = y^*$ in eq. (6.6). We thus obtain;

$$\pi_2 y^{*2} + (\pi_1 - 1)y^* + \pi_0 = 0 \tag{6.12}$$

Solving the above quadratic in y* and substituting the values of π_0, π_1, and π_2 into it from eq. (6.7) yields the following pair of solutions (y*1,y*2) given by:

$$y^*_{1,2} = \frac{[(-\beta_1 G_1 + 2\beta_1 G_2 \bar{y} + 1) \pm \{(\beta_1 G_1 - 1)^2 - 4\beta_1^2 G_2 (G_1 \bar{y} - E_{t-1} m_t)\}^{\frac{1}{2}}]}{2\beta_1 G_2} \quad \ldots(6.13)$$

We now need to determine the condition(s) under which one of these solutions will be the natural rate given by y. In order to do so, we initially substitute y in the left-hand side of eq. (6.13) above yielding:

$$\bar{y} = \frac{[(-\beta_1 G_1 + 2\beta_1 G_2 \bar{y} + 1) \pm \{(\beta_1 G_1 - 1)^2 - 4\beta_1^2 G_2 (G_1 \bar{y} - E_{t-1} m_t)\}^{\frac{1}{2}}]}{2\beta_1 G_2} \quad \ldots(6.14)$$

which yields: $(\beta_1 G_1 - 1) = \pm \{(\beta_1 G_1 - 1)^2 - 4\beta_1^2 G_2 (G_1 \bar{y} - E_{t-1} m_t)\}^{\frac{1}{2}}$ (6.15)

By squaring both sides and simplifying, we obtain the following condition, which states that if the equilibrium level of output is to coincide with the natural rate, then the level of the money stock expected by the public must be equal to:

$$E_{t-1} m_t = G_1 \bar{y} \tag{6.16}$$

The above phenomenon highlights the true implication of the chaotic process under consideration. Because the public views the monetary authorities as a black box, which generates values for yt by an unknown (to them) process, they are unable to from objectively correct assessments of the probabilities.

This implies that they are forced to consider |yt| as a random process whose autocovariance function, given by eq. (4.2), in the same as that of white noise. Thus, by possessing such subjective beliefs regarding the apparent randomness of policy, their best prediction of all values of yt is its mean value which is the natural rate y, i.e.

$$E\{y_t\} = E\{y_{t-1}\} = \bar{y} \qquad (6.17)$$

where E(.) is the expectations operator.

Substituting eq. (6.2) into eq. (6.3) and taking conditional expectations on both sides yields:

$$E_{t-1}m_t = G_1E\{y_{t-1}\} + G_2[E\{y_{t-1}\} - \bar{y}]^2 \qquad (6.1R)$$

which, along with eq. (6.17), yields eq. (6.16).

It is thus evident that the condition derived algebraically from eq. (6.14) is consistent with the strict form of rational expectations: the public understands the new (random) policy and incorporates its (mean) effects into their decisions immediately.

Substituting eq. (6.16) into eq. (6.13) yields the following two solutions for the quadratic equation:

$$y^*_1 = \frac{[(-\beta_1G_1+2\beta_1G_2\bar{y}+1) + \{(\beta_1G_1-1)^2\}^{\frac{1}{2}}]}{2\beta_1G_2}$$

$$= \bar{y} \qquad \ldots(6.19a)$$

and

$$y^*_2 = \frac{[(-\beta_1G_1+2\beta_1G_2\bar{y}+1) - \{(\beta_1G_1-1)^2\}^{\frac{1}{2}}]}{2\beta_1G_2}$$

$$= \bar{y} + [(1-\beta_1G_1)/\beta_1G_2] \qquad \ldots(6.19\text{ b})$$

The important message conveyed by eq. (6.19b) is that The feedback coefficients, G1 and G2, do enter into the determination of output yt: Systematic monetary policy can be effective.

6.3 Stability of Equilibrium

To see exactly how effective it is, we have to ascertain whether or not it can raise the output level beyond the natural rate. Now if *y*2* has to be above the natural rate, then:

$$(1 - \beta_1 G_1)/\beta_1 G_2 > 0 \qquad (6.20)$$

Which, given the assumption in eq. (4.10b) that G2 < 0, implies:

$$G_1 > 1/\beta_1 \qquad (6.21)$$

Which, considering that $\beta_1 > 0$, implies that G1 mush be positive. Eq. (6.21) is consistent with eq. (6.9a) which specifies the restriction on G1 if the model is to generate chaos.

Therefore, if the monetary authorities set G1 so as to satisfy the constraint eq. (6.21), then the second equilibrium solution y*2 will exceed the natural rate y.

Having established that there exists a solution for yt that exceeds the natural rate, we now have to ensure that the system trajectories do converge to it. The necessary and sufficient condition for such convergence is that: (i) the equilibrium solution y*1 is unstable, and (ii) the equilibrium solution y *2 is stable. We must therefore examine the parametric restrictions under which both these above conditions will be satisfied and, in order to do so, we have to estimate the slopes of the phase graph at these two equilibrium solutions.

From eq. (6.11),the slope of the phase graph at y*1 is:

$$\delta y_t/\delta_{t-1} = \beta_1 G_1 - 2\beta_1 G_2 \bar{y} + 2\beta_1 G_2 [\bar{y}]$$
$$= \beta_1 G_1$$

and the slope of the phase graph at y*2 is:

$$\delta y_t/\delta_{t-1} = \beta_1 G_1 - 2\beta_1 G_2 \bar{y} + 2\beta_1 G_2 [\bar{y} + (1-\beta_1 G_1)/\beta_1 G_2]$$
$$= 2 - \beta_1 G_1$$

Given that β_1 and G1 are both positive, the stability analysis in Section 4.2 provides the basis for specifying the restrictions on G1 if the system is to converge towards y*2.

Thus, the equilibrium solution at y*1_ will be unstable if:

$$|\beta_1 G_1| > 1 \tag{6.23a}$$

Which implies that

$$G_1 > 1/\beta_1 \tag{6.23b}$$

Which is seen to be identical to eq. (6.21). Thus, the very restriction which ensures that y*2 > Y also ensures that y*1 will be unstable. Now, for the equilibrium solution y*2 to be stable:

$$|2-\beta_1 G_1| < 1 \tag{6.24a}$$

Which implies that:

$$3/\beta_1 > G_1 > 1/\beta_1 \tag{6.24b}$$

Therefore, if eq. (6.24b) is satisfied, then, both, y*1 will be unstable and y*2 will be stable which implies that the system will converge to the rate which is higher than the natural rate.

Thus, it is seen that both the feedback coefficients in the policy rule have an important role to play: G1 and G2 together determine the equilibrium solution y*2' while G1 determines the stability of this equilibrium solution. It is in this senate therefore that systematic monetary policy can be effective.

We shall now, using a simple numerical example to model rational expectations, probe further into the dynamics of the resulting policy effectiveness proposition in order to demonstrate that, as there are no possible paths from one steady state to another, it would be impossible for the public, in the presence of such chaotic trajectories, to adjust to any change in regime and thereby model a learning process.

7. A Numerical Illustration

Consider the following numerical illustration where we set: β_1 = 0.5; G1 = 5.5; G2 = -0.25; and y = 1. Following eq. (6.16), we set Et-1mt= G1y = 5.5. Therefore, eq. (6.1) is given by:

$$y_t = 0.5m_t - 1.75 \qquad (7.1)$$

While eq. (6.2) will be equal to:

$$e_t = (y_t - 1) \qquad (7.2)$$

And the feedback control law, eq. (6.3), will be specified as:

$$m_t = 5.5y_{t-1} - 0.25e_{t-1} \qquad (7.3)$$

The ensuing chaotic process, given by eq. (6.6), is obtained by initially substituting eq. (7.2) into eq. (7.3) and the result into eq. (7.3). Thus, our logistic model is given by:

$$y_t = -1.875 + 3y_{t-1} - 0.125y^2_{t-1} \qquad (7.4)$$

The slope of the phase graph is given by:

$$\delta y_t/\delta y_{t-1} = 3 - 0.25y_{t-1} \qquad (7.9)$$

The two equilibrium solutions of the chaotic system are:

$$y^*1 = 1 \qquad (7.10a)$$

and

$$y^*2 = 15 \qquad (7.10b)$$

As $3/\beta_1$ >G1 >1/p1, eq.(6.24b)is satisfied, implying that Y*1 will be unstable, and y*2 will be stable. To show this, we substitute the above solutions into eq. (7.9) yielding:

$$\delta y_t/\delta y_{t-1}\Big|_{y^*_1=1} = 3 - 0.25(1) = +2.75 \qquad (7.11a)$$

$$\delta y_t/\delta y_{t-1}\Big|_{y^*_2=15} = 3 - 0.25(15) = -0.75 \qquad (7.11b)$$

The phase graph of the above chaos model is given in Figure 1. The resulting trajectory for yt obtained by setting mo = 30 and simulating from t=0 to t=100 is given in Figure 2.

While the above model has been solved by assuming:

$$E_{t-1}m_t = \mu G_1\bar{y} = 5.5 \quad (7.12)$$

with a setting of u = 1, it would be interesting to determine how these two equilibrium solutions would change, given changes in expectations. Simulating the chaotic system for $0 \leq \mu \leq 3$, we obtained alternative solutions for μ which have provided in Figure 3 (which plots values of μ on the x-axis and the corresponding equilibrium solutions of yt on the y-axis).

The results indicate that for $\mu < 0.4$, the system undergoes a bifurcation which yields a 2-period limit cycle. However, for $\mu \geq 0.4$, the system converges to a single equilibrium solution which decreases steadily with rising expectations, following the logic of the monetary-surprise model. However, no matter how high expectations are raised, the system never regains its natural rate, implying that monetary policy can be effective even if the public is reacting to monetary surprises by continuously updating its objective assessments of the probabilities.

However, it seems natural to assume that it would be far more difficult to correctly form objective assessments of the probabilities if the public possesses subjective beliefs regarding the apparent randomness of policy.

In order to prove this point, we now set G1 = 7.4 in the chaos model (all other parameters remaining unchanged). This would imply that with μ = 1 in eq. (7.12), we would have:

$$E_{t-1}m_t = y\ G1y = 7.4 \quad (7.13)$$

The resulting chaotic trajectories for yt and mt obtained by simulating the nonlinear system between time periods 200-300 and 0-300, respectively, are given in Figures 4 and 5.

Figure 1

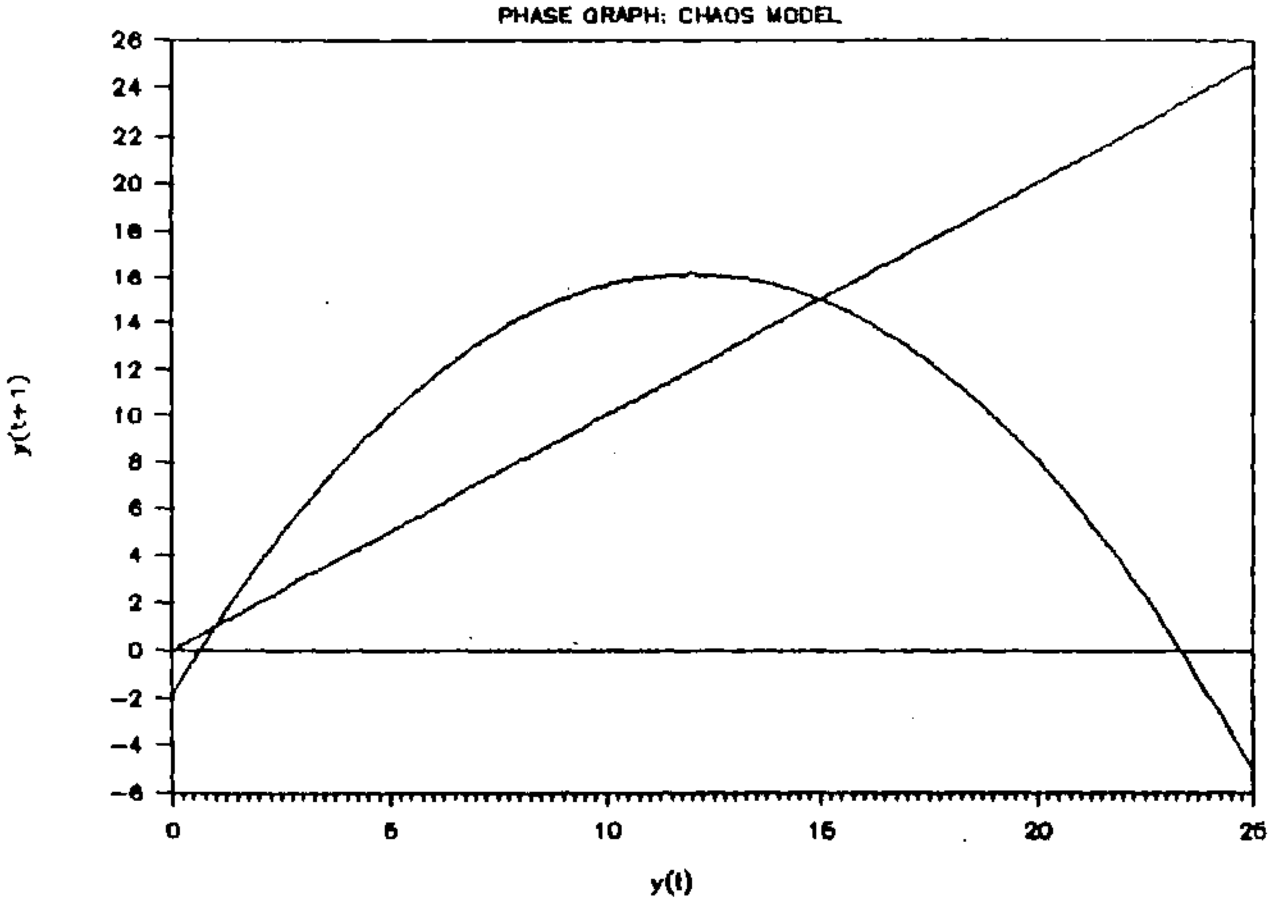
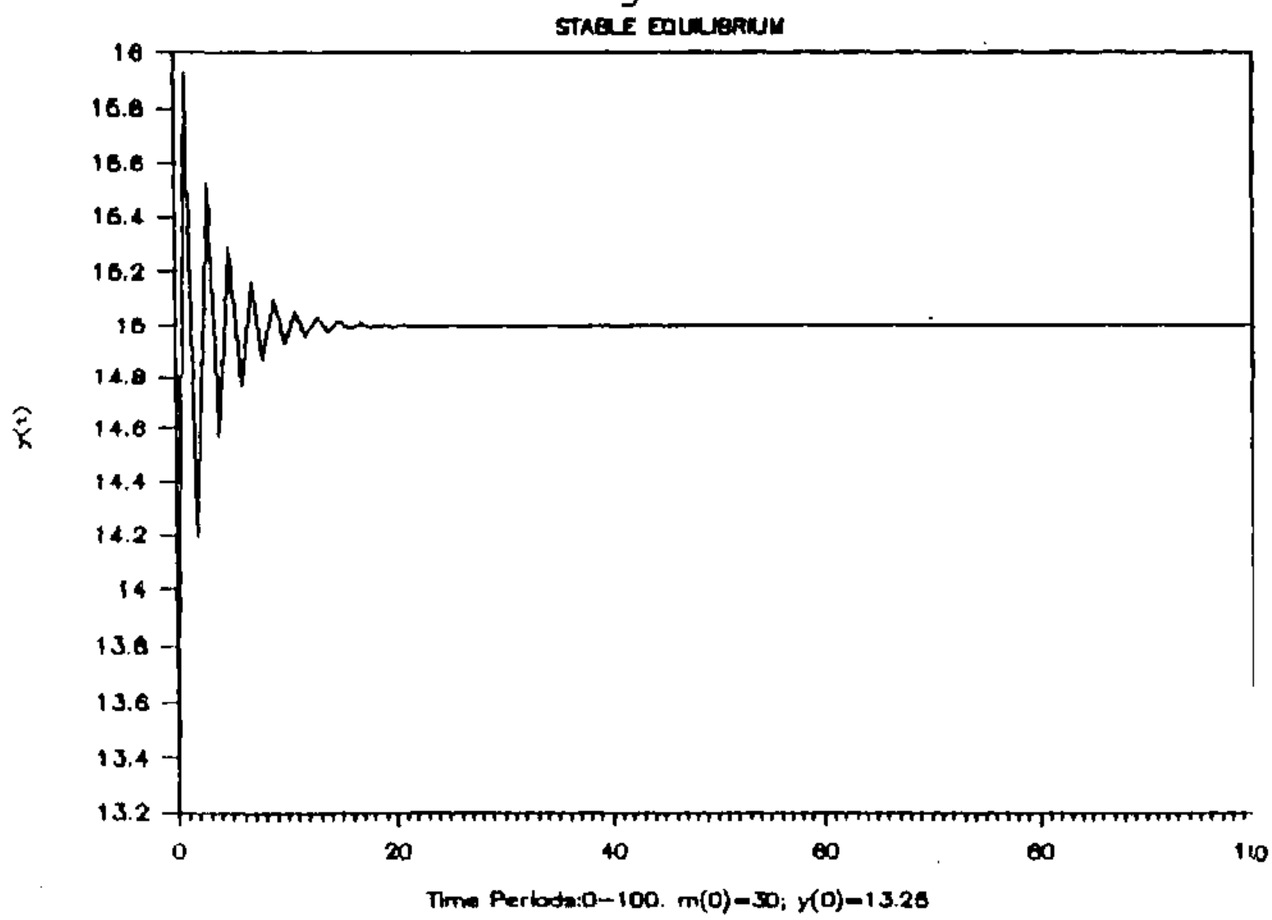


Figure 2



As before, we again determine how these chaotic solutions would change, given changes in expectations. Simulating the chaotic system for $0 < \mu \leq 1$, we obtained alternative patterns of behaviour for yt which have provided in Table 1 below.

Table 1

BEHAVIOUR UNDER ALTERNATIVE ASSUMPTIONS REGARDTNG EXPECTATIONS

V	Pattern of yt	Equilibrium Solution(s)
0.1	Chaotic	-
0.2	4-period cycle	1.172; 4.594; 15.642; 31.337
0.3	Chaotic	-
0.4	Chaotic	-
0.5	Chaotic	-
0.6	3-period cycle	5.154476; 15.69410; 29.85859
0.7	8-period cycle	26.265; 20.879; 7.500; 29.063; 17.647; 6.066; 29.489; 15.710
0.8	Chaotic	-
0.9	Chaotic	-
1.0	Chaotic	-

The results clearly indicate that unlike the earlier (stable) case when the equilibrium solutions bifurcated smoothly when u was reduced below a critical level, in the current (chaotic) case, chaos and limit cycles occur randomly as expectations range from $0 < \mu \leq 1$. It is very interesting to note that, at $\mu = 0.6$, a 3-period cycle - which is one of the rarest possible phenomena to be observed empirical1y - manifests itself. Thus, it is seen that, under an apparently randomly generated policy, it would be far more difficult to correctly form objective assessments of the probabilities.

Figure 3

EXPECTATIONS AND EQUILIBRIUM: STABLE

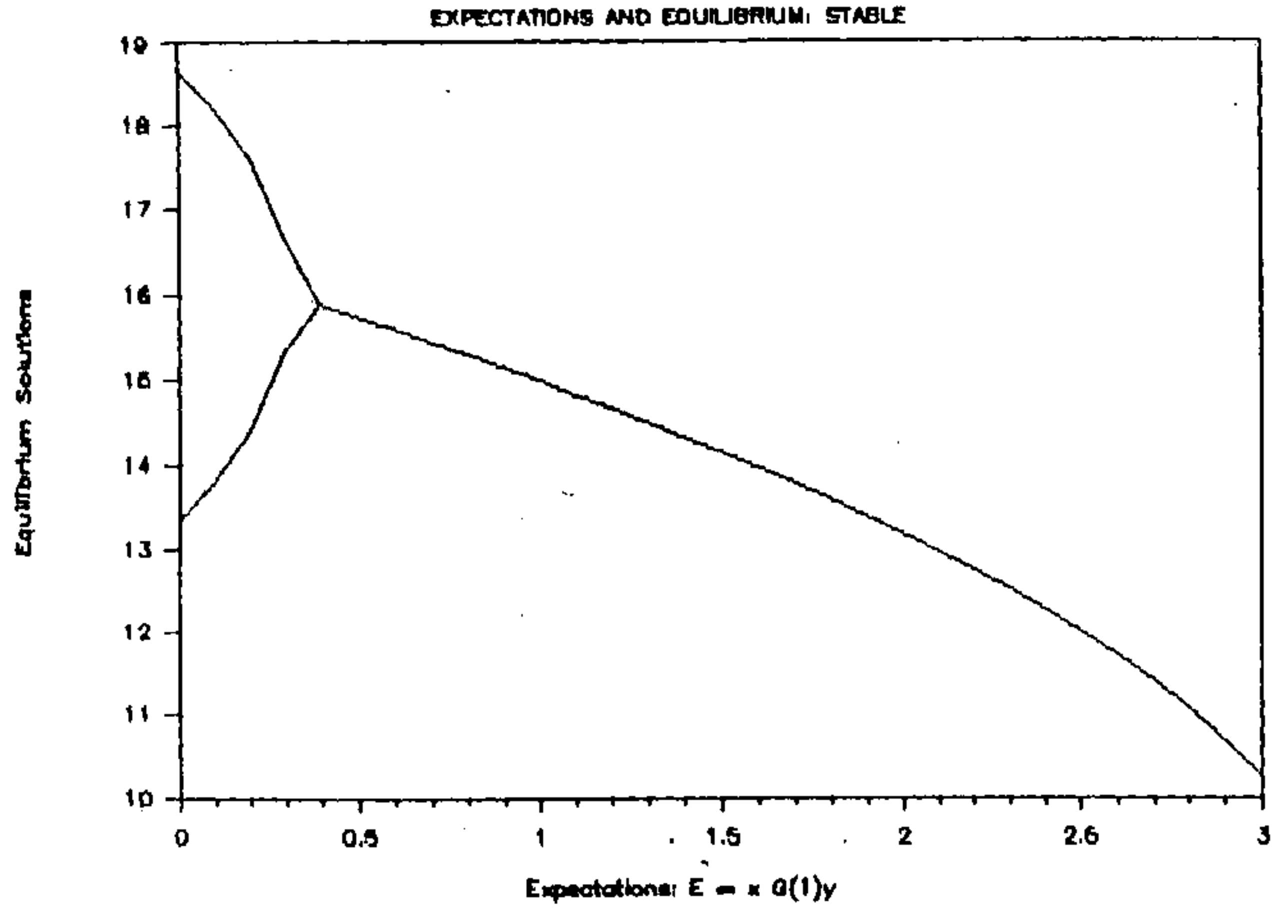
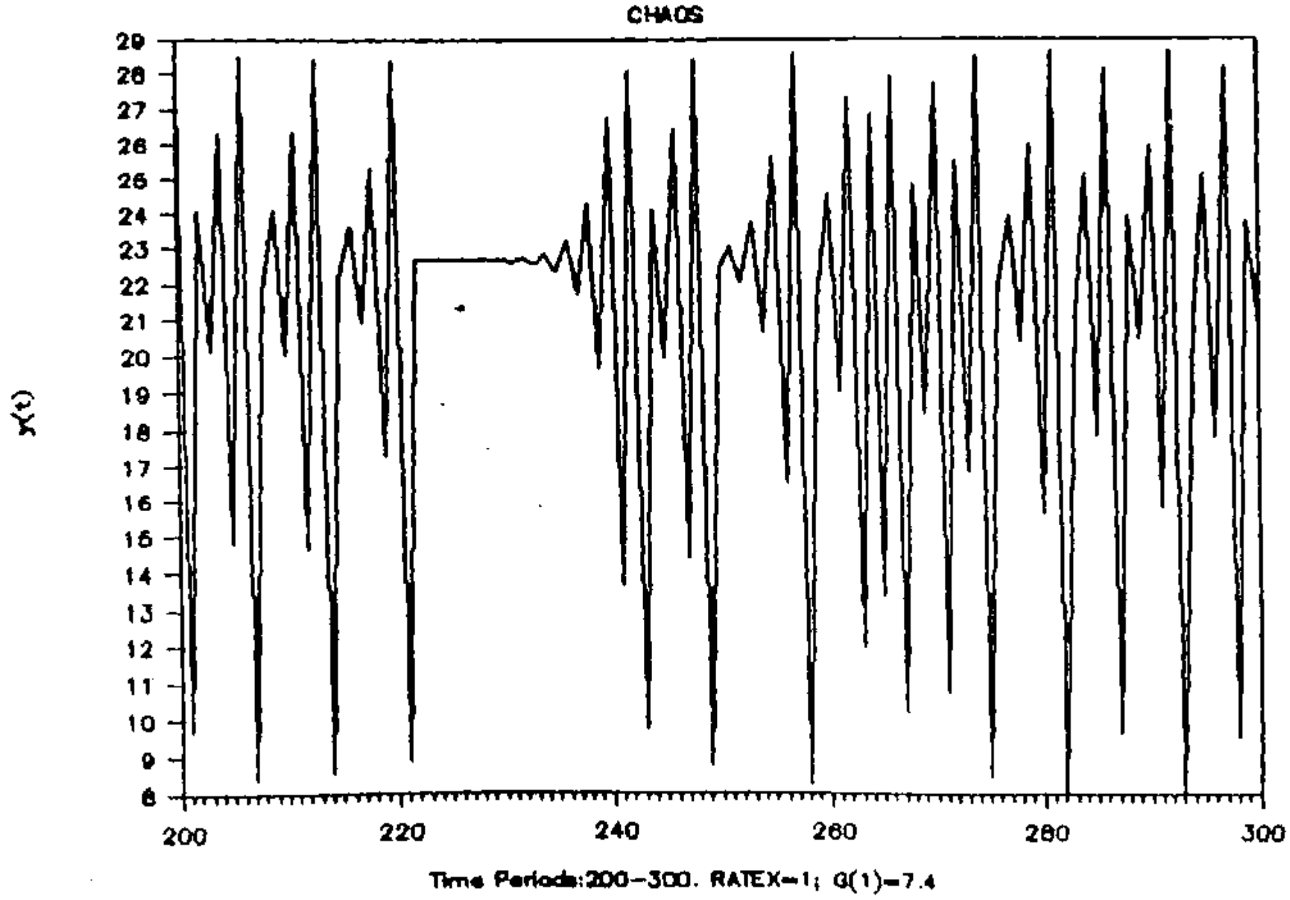


Figure 4

CHAOS



This point can be made more forcefully if we consider the results of the following experiment where we varied μ by small increments around the region, which corresponds to that of rational expectations. We observe that for values of μ close to unity, there is chaos and, hence, we use the mean of the chaotic process as an indicator of whether it would be possible to model the learning process. The results are displayed in Figure 5 where we have simulated the chaotic system between $0.9 \leq \mu \leq 1.1$ and, for each value of μ, we have the corresponding mean of the chaotic process estimated over the time periods t=200 to t =300

The results are very interesting because they suggest that as the mean of such a chaotic process would be an apparently random phenomena with no obvious pattern, people would be unable to update their subjective probabilities on the basis of their past mistakes and hence it would be impossible for them to model the learning process. Thus, even the weaker form of rational expectations is seen to be unsubstantiated.

Moreover, in such a case, as the economy either never settles down to a steady state or, if it does (like in Figure 4, where between periods 222 to 235, the economy hovers practically motionless at 22.69 thereby simulating a "virtual steady-stare"), does so only momentarily, the public would never be able to understand the characteristics of any policy because new events neither have any precedent nor do they bear any resemblance to previous patterns. Consequently, they would never be able to adjust to a change in regime, thereby pre-empting even the strong version of rational expectations. Thus, the policy ineffectiveness proposition is seen to be vitiated.

Figure 5

CHAOTIC MONETARY POLICY

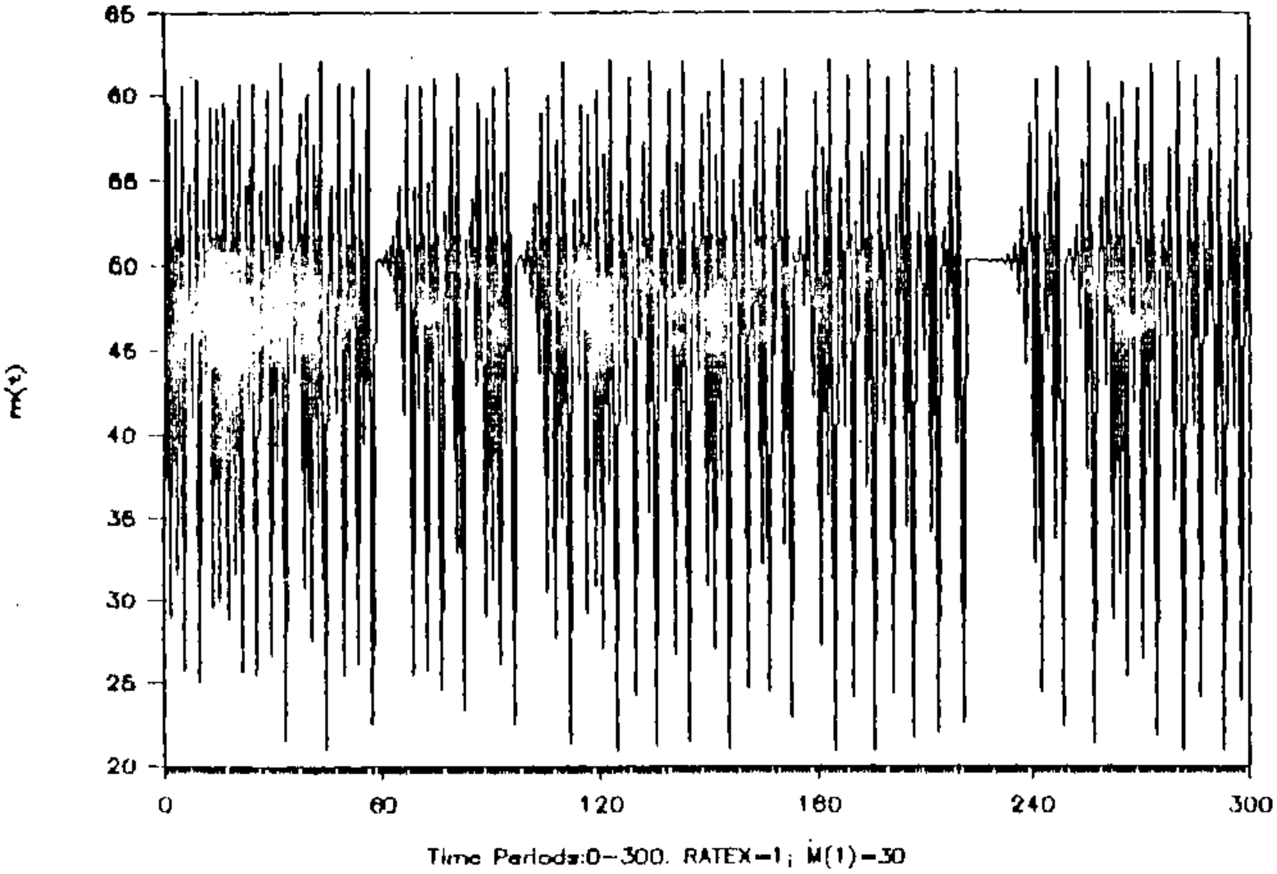
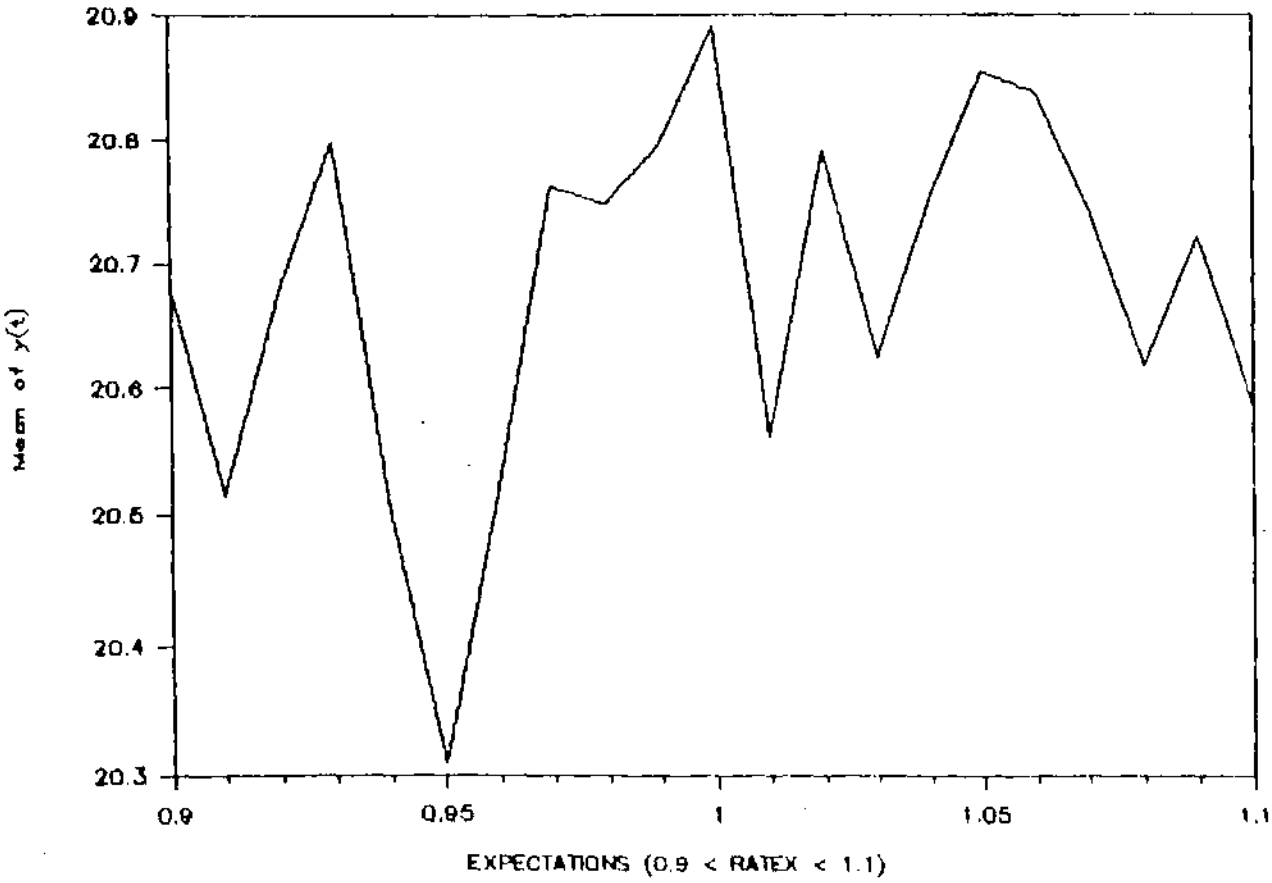


Figure 6

EXPECTATIONS AND CHAOTIC MEANS



4. Conclusion

In this paper we have shown that it i a possible to specify a control model incorporating a feedback law of the PTD-variant which yields reduced form estimate of the system from which the parameters of the control model are recoverable, thereby enabling the econometrician to estimate controlled behavioural equations.

Ft is also seen that when a monetary surprise model in represented •in terms of such a control system, the coefficients in the feedback rule enter into the determination of output, thereby proving that systematic monetary policy can be effective.

The results indicate that under certain parametric restriction the system generates either: (i) a stable equilibrium solution for output (which is higher than the natural rate); or (ii) chaotic trajectories for output.

It is shown that as long as the system is stable, increasing expectations does reduce the equilibrium level of output although these output solutions are invariably more than the natural rate. However, and more importantly, it is seen that increasing or decreasing expectations when the system is within a chaotic regime changes the mean of the chaotic process in an apparently random fashion implying that it would be impossible to model a learning process based on the irregular patterns being observed. Thus, both the weak as well as the strong versions of the rational expectations hypothesis are seen to be unsubstantiated.

In concluding, we hope that the results presented in this paper which do serve to undermine the Lucas critique restore some legitimacy to the use of control theory and feedback rules when applied in the context of macroeconometric policy evaluation.

REFERENCES

Barro, R.J. and D.R. Gordon (1983a), 'A positive theory of monetary policy in a natural rate model Journal of Political Economy, 91*(A), 569*-610.

Barro, K.J. and O.B. Gordon (1983b), Rules, discretion and reputation in a model of monetary policy', Journal of Monetary Economic, 12(2), 101-21.

Baumol,W.J. and J.Betrabib (1989), Chaon: Significance, mechanise and economic applications', Journal of Economic Theory,26(3) 318-39 .

Bray, M. (1982),learning, estimation and the stability of rational expectations'. Journal of Economic Theory, 26(3), 318-39.

Bray, M, (1983), Convergence to rational expectations Equilibrium', in R. Frydman and E.S. Phelps (EON.) Individual Forecasting and Aggregate Outcomes, Cambridge University Press, Cambridge.

Brock, W.A. (1986), 'Distinguishing random and determinister systems: Abridged version', Journal of Economic Theory. 40. Cooley,T.F.S.F. LeRoy and N. Raymon (1984a), 'Econometric policy evaluation: Note", American Economic Review, 74(3), 467-70.

Cooley, T.F., S.F. Leroy and N. Raymon (1984b), 'Modeling policy interventions', Unpublished typescript, University of California, Santa Barbara, and University of Missouri Columbia.

Hoover, K.D. (1988), The New Classical Macroeconomics, Rasil Blackwell, Cambridge, Mass.

Li T.Y. and J.A. Yorke (1975), 'Period three implies chaos' American Mathematical Monthly, 82.

LucaH, H.R, Jr. (1976), 'Econometric policy evaluation: A critique, in K. Brunner and A.H. Meltzer (Eds) The Phillips Curve and labor Market, North-Holland, Amsterdam.

Lucas, H.E., Jr. (197[7]), 'Understanding business cycles', in K. Brunner and A. H. Meltzer (Eds.) Stabi1ization of the Domestic and International Economy, North-Holland, Amsterdam.

Lucaa, R.E, Jr. (1905), 'Adaptive behaviour and economic theory' Journal of Business, 59(4), S40I-26.

Rao, M...S. vl990), 'The role of feedback in macroeconomic policy, in R. Trappl (Ed.) Cybernetics and Systems '90, World Scientific Publishing Company, Singapore.

Sargent, T.J. (1984), ' Autoregreasiona, expectation and advice', American Economic Review, 74(2), 408-15

Sims, C.A. (1980), 'Macroeconomics and reality[1], Econometrica, 48(1), 1-48.

Sims, C.A. (1982), 'Policy analysis with econometric models', Brookings Papers on Economic Acti vi ty, (1), 107-52.

Sims, C.A. (1986), 'A rational expectationa framework for shore run policy analysis', in W.A. Barnett and K.J. Singleton (Eds.) New Approaches to Monetary Economics; Proceedinqs of the Second International Symposium in Economic Theory and Econometrics, Cambridge University Press, Cambridge.

Taylor, J.B. (1975), 'Monetary policy during a transition to rational expectations' Journa1 of Po1itica1 Economy, 83(6), 1009-21 .