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A MODIFIED HANNAN INEFFICIENT PROCEDURE FOR CAUSAL SYSTEMS

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1 INTRODUCTION

A. Hannan Inefficient Procedure

An important approach to the analysis of distributed laq **systems** in economics stems from the seminal papers of Hannan (1963,1965,1967) based on multivariate spectral methods.The **three Major** characteristics of **these** frequency **domain methods are**:

(1) No constraints are placed on the coefficients of the **distributed** lags (as in the more traditional methods of **Koyck (1954),Dhrymes (1971),Almon** (1965) etc).

(2) The residual terms need not be serially independent and

(3) **The length of the distributed lag** need **not be specified a priori**.

The discrete time model considered by Hannan **was** of the form

$$Y(t)=\sum_{j=-p}^{q} b(j)\ X(t-j)+e(t) \qquad (!)$$

Hannan assumed Y(t) and X(*t)* to be real vector **series which were** jointly covariance-stationary. Me however choose to work with univariate Y(t) and X(t) (assumed jointly covariance **stationary)** though the results we prove in **this** paper have **straight forward** generalisations to the vectorial case.The continuous time version of (1) is of course

$$Y(t) = \int_{-\infty}^{\infty} b(\lambda) X(t-\lambda) d\lambda + e(t) \quad (2)$$

where $b(\lambda)$ may vanish outside a finite range (c,d).

By taking expectations and then Fourier transform we get the identity

$$f_{yx}(w) = H(w) f_{xx}(w) \quad (3)$$

where fxx (w) is the autospectruro of X(t) and Fyx(w) is the cross-spectrum between X(t) and Y(t) and H(w> is the frequency response (or transfer) function. Letting the """ superscript stand for estimates , a suggested estimate for $b(\lambda)$ in (2) would be

$\hat{b}(\lambda) =$ Inverse Fourier Transform of H(w)=s exp(-iwx) H(w) dw
—CD

$$= \int_{-\infty}^{\infty} \exp(-iw\lambda) \{\hat{f}_{yx}(w) / \hat{f}_{xx}(w)\} dw$$

For the discrete case the corresponding expression is

$$\hat{b}(j) = (1/2M) \sum_{k=0}^{M} \delta(k) \hat{H}(w_k) \exp(-ijw_k) \quad (5)$$

	=1	k=0 and M
where &(k)	=2,	k-1,2. M-l
	=0	otherwise

where M is the truncation parameter used in the estimation of $\hat{f}_{xx}$ and $\hat{f}_{yx}$.

The coefficients yielded by (4) and (5) are called Hannan "Inefficient" estimates.

The general conditions for the upholding of the above results are those given by Grenander and Rosenblatt(1957),(Chapter 7). The estimators are inefficient as their variances computed from

$$\operatorname{Var}(\hat{b}(j)) = (1/2TM) \sum_{k=0}^{M} \delta(k) [\hat{f}_{ee}(w_j) / \hat{f}_{xx}(w_j)] \qquad (6)$$

are larger than those of the Best Linear Unbiased estimator . In (6) T is the length of the record and

$$\hat{f}_{ee}(w_j) = [\hat{f}_{yy}(w_j) - \hat{f}_{xx}(w_j)\hat{H}(w_j)]$$

Note that (6) pertains to the estimators (5) of the discrete system Hannan (1967) has derived "efficient" estimators too.For the system (1) these are defined as

$$\hat{b} = \hat{K}^{-1} \hat{d} \qquad (7)$$

where the elements of K are

$$\hat{K}_{rs} = \sum_{j=-M+1}^{M} \hat{f}_{ee}^{-1}(w_j)\hat{f}_{xx}(w_j) \exp(iw_j(r-s)) \qquad r,s=-p\ldots q \qquad (8)$$

and the elements of d are

$$\hat{d}_r = \sum_{j=-M+1}^{M} \hat{f}_{ee}^{-1}(w) \hat{f}_{xy}(w_j) \exp(iw_j r) \qquad <9>$$

The efficient and inefficient estimators coincide if the signal to noise ratio $\{ f_{xx}(w_j)/f_{ee}(w_j) \}$ is constant across the frequencies
wj.(see Hannan (1967).

B. Limitationsof Hannan's Method

There is one important drawback of Hannan's method. The transfer function H<w) defined by (3) presumes that the output series {Y(t) is affected by both past and future values of the input series {X(t)}, This means that the system is non-causal or physically unrealisable. Many systems in economics are causal with the unidirectional causality established either from a priori considerations or from the execution of the Granger-Sims type causality tests (Granger(1969), Sims<1972>).

For such cases the lower limits in the summation (1) or the integral (2) are O.It is a common practice among economists to apply the Hannan's inefficient procedure in this case simply by setting $b(\lambda)=0$ for $\lambda<0$ in the continuous cane (4) (see RATS version 2.0 for example) or by using (5) without modification in the discrete case (see Cargil} and Meyer (1972).

One of the very few analysts to explicitly recognise the inappropriateness of this procedure is Whittle(1963). He hinted at the removal of this anomaly using the classical Wiener-Hopf factorisation but did not present an explicit method applicable in practice. The main problem with the use of the Wiener-Hopf technique in this context are two fold

(1) Firstly except in the most elementary cases the spectral and cross spectral density functions will rarely be given explicitly as functions of the frequency

(2) Even if they were so qiven the Wiener-Hopf factorisation is extremely difficult to achieve in practice (Noble (1950)). *In* this paper we suggest a method for the problem of estimating the (one sided) distributed lag coefficients in a causal system by a modification of Hannan's procedure.Three advantages are claimed for our method

(1) it is much easier to implememt in practice as compared to the Wiener-Hopf method

(2) it does not require the spectral densities to be explicitly expressed as functions of the frequency parameter w and

(3) it can be generalised in a straightforward manner to the case where {Y(t)} and (X(t)l are non stationary via appeal to Priestley's (1965) concept of the evolutionary spectrum. The plan of this paper is as follows. The next section discusses our method for the continuous case. Sections 3 and 4 are concerned with the estimators in the discrete case and their distributions. Section 5 presents some simulation evidence whereas Section 6 considers the extension of our method to the nonstationary case. Conclusions are gathered in Section 7.

2. CONTINUOUS CAUSAL MODELS

A.The_Mellin_Integral

The general continuous regression model is as described by (2). We assume that {Y(t)} and {X(t)are jointly covariance stationary and that {e(t)l Is a white noise process

We define' $b_+(\lambda)$ and $b_-(\lambda)$ as follows (Noble (1950)).

$$b_+(\lambda) = b(\lambda),\ \lambda\geq 0 \qquad b_-(\lambda) = b(\lambda),\ \lambda<0$$

$$b_+(\lambda) = 0,\quad \lambda<0 \qquad b_-(\lambda) = 0,\quad \lambda\geq 0$$

Let H (w) and H_(w) denote the Fourier transforms of $b_+(\lambda)$ and $b_-(\lambda)$ so that $H(w) = H_+(w) + H_-(w)$ is,as before,the transfer function of the system (2).

If the system (2) is causal/physically realisable then $b_-(\lambda)$ = 0 for all λ and $b(\lambda)$ =b. (λ) for all λ. Thus H<w)=H (w). Hence in dealing with causal systems, the estimates $b_+(\lambda)$ can be obtained (analogously to the Hannan method) from the empirical estimate of $H_+(w)=H(w)$

Now $$H_+(w)=\int_0^\infty \exp(i\lambda w)b_+(\lambda)\ d\lambda \qquad 10)$$

put w=ip in (10) so that

$$H_+(p)=\int_0^\infty \exp(-p\lambda)\ b_+(\lambda)\ d\lambda$$

(11) means of course that $H_+(p)$ is the Laplace transform of $b_+(\lambda)$ (see Sneddon (1951 >,Pearson (1903) etc). The Laplace transform will exist under the following conditions.

(a) $b_+(\lambda)$ has on every finite interval of the λ-axis at most a finite number of discontinuities of the first kind

(b) 3 non negative constants M and ex such that

$$|b_+(\lambda)|\leq M\exp(\alpha\lambda) \qquad (12)$$

We assume that the b (X.) are bounded so that α= 0 in (12) and that condition (a) is valid. Under these conditions (11) is defined for Re(p) >0 (or Im(w)>0) and $b_+(\lambda)$ can be obtained by the well known Mellin's Inversion Formula (See Morse and Feshbach (1953), Churchill (I960)).

$$b_+(\lambda) = (1/2\pi i) \int_{x-i\infty}^{x+i\infty} \exp(p\lambda) H_+(p)\, dp \qquad x>0 \qquad (13)$$

Integral (13) exists provided the following conditions are satisfied

(a) $H_+(p)$ is an analytic function in the domain Re(p) >0 (i$_e$ Im(w) >0)

(b) $H_+(p)$ tends to zero uniformly in arg(p) as :p : tends to ooand is bounded.

(o) For all Re(p)>0 the real integral

$$\int_{x-i\infty}^{x+i\infty} |H_+(p)|\, dy, \qquad x>0$$

is bounded.

We assume once again that the above conditions are fulfilled. Note that the integrals (13) and (14) are calculated along the straight line Re(p) ■ x and are taken in the sense of the principal value. For example, (13) is defined as

$$\lim_{A \to \infty} (1/2\pi i) \int_{x-iA}^{x+iA} \exp(p\lambda) H_+(p)\, dp$$

It is a well known result in complex analysis that the integrals (13) or (14) do not depend on the choice of x (as long as x>0).

B.computational Consideration

The evaluation of the Mellin integral (13) ia an extensively researched field (see eg Erdelyi (1954) ,Krylov (1962) and Pearson(1903))» Two cases may be distinguished -first where $H_+(p)$ is qiven explicitly as a function of p and second, where it ia not so qiven. Most cases considered in practice naturally fall into the second category.

Case 1 ($H_+(p)$ qiven explicitly as a function of p)

We present below a result due to Doetech (1956) adapted to the terminology of our problem.

Theorem (Doetech):If $H_+(p)$analytic at the point at infinity (OD) and in the neighbourhood of QD has a Laurent expansion

$$H_+(p) = \sum_{k=1}^{\infty} (c_k p^{-k}) \text{ , then}$$

$$b_+(\lambda) = \sum_{k=1}^{\infty} [c_k /(k-1)!]\lambda^{k-1} \qquad (15)$$

Case 2: $(\hat{H}(p)$ not expressed as a function of p)+

Doetsch theorem while very powerful in the derivation of theoretical results is rarely suitable for applications since-$H_+(p)$ will rarely be explicitly expressed as function of p. \ In practice the values of the transfer function estimate $\hat{H}_+(w)$ will be available at a finite number of points say w_0 w1. ..... wm which without loss of generality, we will suppose to be equidistant. We first make the substitution

$p=iw$ to obtain $\hat{H}_+(p)$.

The corresponding points are then $p_0, p_1 ... p_m$

The approximation method that we present is due to Krylov and Skoblya (1977).

Define Define $l_k(1/p)$ by

$$l_k(1/p) = \prod (p_i^{-1} - p_i^{-1}) \; [\prod (p_k^{-1} - p_i^{-1})]^{-1} \tag{16}$$

The index i in (16) ranges over all integers from 1 to m ,with the exception of k.

Theorem (Krylov and Skoblya) Suppose $H_+(p)$ is regular in the half plane $Re(p)>o$ and let p_0 p. p be a set of

equispacedpoints with $Re(p,)>2a, j=0,1,...m$ then $b_+(\lambda)$ can be approximated as

$$b_+(\lambda) \cong \sum_{k=0}^{m} \{ \sum_{j=0}^{m} a_{kj} \lambda^{j-1} [\Gamma(j)]^{-1} \} H_+(p_k) \tag{17}$$

where a_{kj} is the coefficient of $(1/p^j)$ in the expansion of 1. (l/p) in powers of <l/p) . The approximation converges uniformly to $b_+(X)$ as m tends to ».

The assumptions of the above theorem are easily satisfied for most $H_+(p)$ arising in economics. In the rare case where the function $H_+(p)$ fails to exist or be continuous (say because• of a jump in the spectral density of (X(t)) the points P_n p,........p should be taken sufficiently to the right of the discontinuity point. The functions (13) and (17) are easily computable.Of course $\hat{b}_+(\lambda)$ is estimated by replacing

$H_+(p_k)$ by its estimate $\hat{H}_+(p_k)$.

3, DISCRETE CAUSAL MODELS

Consider now the discrete model (1) which we rewrite in the snore general form

$$Y(t)=\sum_{j=-\infty}^{\infty} b(j)\ X(t-j)\ +e(t) \quad (10)$$

subject to the same assumptions as in the continuous case. Our interest is in the causal system for which b(j)=O for 1<0. Whittle (1963) has suggested that for such a causal system b<1> (j=0,1,2..) is given as the coefficient of w^ in the following expression

$$\{1/\sigma^2 G(w)\}^{-1}[f_{yx}(w)/G(w^{-1})]_+ \quad . \qquad (19)$$

where

(l)fyx(w) is the cross spectrum between (Y(t)} and (X(tU)

(2) G(w) is the canonical factorisation of the spectral density function fxx (w)

(3) σ^2=var (e(t)> and

(4) the subscript ♦ indicates that the bracketed summation is performed over non negative indices only.

The canonial factorisation of fxx (w) has to be based on the following theorem due to Doob (1953) p.160.

Theorem (Doob): spectral density function $f_{xx}(w)$ satisfies the condition

$$\int_{-\pi}^{\pi} \log\{f_{xx}(w)\}dw > -\infty \quad (20)$$

Then ∃ a unique sequence s_0, s_1, s_2………… (with q_0 real and positive) and $\sum_{u=0}^{\infty} |q_u^2| < \infty$ such that the function

$$G(w) = \sum_{u=0}^{\infty} q_u w^u \quad (21)$$

has no zeros in the reqion 0< :w :<1 and provides a factorisaion for fxx <w) of the form

$$f_{xx}(w) = \exp(c_o)\ G(w)\ G(w^{-1}) \quad (22)$$

where $c_0 = (1/2\pi)\int_{-\pi}^{\pi} \ln\{f_{xx(w)}\}dw$ and G(0)=1

The actual evaluation of G(w) can be done alonq the lines suqqested by Priestley (1901) p.734-735. 'Assume that ln(fxx(w)) is analytic in an annulus $\rho<|w|<1/\rho,\ (\rho<1)$ and that in this annulus it has *a*
Laurent expansion

$$\ln\{f_{xx}(w)\} = \sum_{k=-\infty}^{\infty} c_k w^k \quad (23)$$

The required factorisation of fxx(w) is now provided by G(w)G(w -1) where

$$G(w) = \exp\{\sum_{k=1}^{\infty} c_k w^k\}$$

with the coefficients ck beinq determined by

$$c_k = (1/2\pi) \int_{-\pi}^{\pi} \exp(iwk) \ln\{f_{xx}(w)\} dw \quad (25)$$

The estimators $\hat{b}(j)$ (for the causal system (10) in which b(j) = 0 i< 0) are obtained by replacing entities in (19) by their estimates.

4.DISTRIBUTIONAL CONSIDERATIONS (DISCRETE CASE)

The distribution of b(j) in (10) when two sided lags are present has been analysed by Hannan (1967), Wahba(1969) and Brillinger (1975). We now derive the distribution of b(j) for one aided or causal systems by adapting some of their results. Our arguments are developed for the discrete case though the continuous case can be analysed analogously. Following Wahba (1969) we make the following assumptions (All The series (X(t)} and (Y(t)} are jointly covariance stationary and normal.

(A2) Let M be the bandwidth parameter of the windows used for the spectral density estimation. Then (M/T)(ln M)—>0 as T—>CD where T is the length of the record.

Under the above assumptions Goodman(1957) and Wahba (1966) demonstrate that:

(1) {H(w)-H(w)} converges in distributions to

$$(2M+1)^{-1}[W_1^c(2M+1, f_{yx}(w)][W_1^c(2M+1, f_{xx}(w)]^{-1} \quad (26)$$

where $W_r^c(n,\Sigma)$ denotes the complex Wishart distribution of dimension r, degrees of freedom n and var-cov matrix Σ (see Srivastava (1965) and Miller (1969)) **and** the other quantities are as defined beforehand

(2.) the estimates FKw^) and $H(w_2)$ for w_1 w_2 are independent We **now** turn to the derivation of the distribution of the estimators based on (19). Define

$$\hat{f}_{ee}(w)=\hat{f}_{yx}(w)\{\hat{f}_{xx}(w)\}^{-1}\hat{f}_{xy}(w) \tag{27}$$

Khatri **(1965) has shown** that the joint density $g(\hat{f}_{ee},\hat{H}_1,\hat{f}_{xx})$ can for every **w** can be factored a« q-F1x F2xF,where

$$F_1 = [\Gamma_1(n-1)]^{-1}|n\hat{f}_{ee}|^{n-2}|f_{ee}|^{1-n} \exp\{-f_{ee}^{-1}(n\hat{f}_{ee})\} \tag{20}$$

$$F_2 = \pi^{-1}|n\hat{f}_{xx}||f_{ee}|^{-1} \exp\{-(\hat{H}-H)^2(n\hat{f}_{xx})\ f_{ee}^{-1}\} \tag{29}$$

$$F_3 = [\Gamma_1(n)]^{-1}|n\hat{f}_{xx}|^{n-1}|f_{xx}|^{-n} \exp\{-n\ \hat{f}_{xx}\ f_{xx}^{-1}\} \tag{30}$$

where n = (T/2M) and $\Gamma_r(n)=\pi^{\{r(r-1)/2\}}\Gamma(n)\Gamma(n-1)\ldots\Gamma(n-r+1)$

From (20) -(30) it follows that f_{ee} is distributed independently of H **and** fxx. This fact together with the observation that F3 is **the** complex Wishart distribution of fxx viz $W_1^C(n,f_{xx})$ means that F2 is the conditional distribution of H qiven fxx which is seen to be multivariate normal. We can now deduce from this in a straight forward fashion that the distribution of the estimators b(j) baaed on (19) are asymptotically normal with mean

$$M^{-1}\{\sum_{v=1}^{M} H(2\pi v/M)\exp(-2\pi ivj/M)\}$$

(31) Further co $\hat{b}(s),\hat{b}(t)\} = n^{-1}M^{-2}\sum_{j=1}^{M} f_{ee}(2\pi j/M)\ \hat{f}_{xx}(2\pi j/M)$

(32)

The above results imply that the hypothesis testing can be done in this case without much difficulty. Note that we could have dispensed with the assumption of normality of X(t) and Y(t) provided we replace (A2> by the stronger requirement (see Hannan (1965))

(A2') (M^2/T) _ >0 and $T(M)^{-4(\tau+1)}$ ---->0 as $T \longrightarrow \infty$

where r>0 depends on the window used for obtaining the spectral estimate.

5.SIMULATION R.SULTS

It was thought that a simulation exercise would throw light on the properties of the estimators that we have proposed in this paper. Henceforth we will call these as modified Hannan estimates. Carqill and Meyer (1974) had conducted an extensive simulation exercise to compare the performance of OLS, Alroon lag and Hannan . inefficient estimators. We use their models 1 and 2 to maintain comparability with their analysis and apart from thethree methods considered by them also consider the method suggested here.

Model 1:

$$y(t)=6x(t)+10x<t-1)+4x<t-2)-1x<t-3)-3x<t-4>-0.5x<t-5)+u(t)$$

$x<t)$ and $u(t)$ are IN(0,1)

Model 2:

$$y(t)=0.5x(t)+1x(t-1)+2x(t-2)+1x(t-3)+0x(t-4)+6x(t-5)+4x(t-6)$$
$$+2x<t-7)+1x(t-0)+0.5x(t-9)+u(t)$$ $x(t)$ and $u(t)$ are IN (0,1)

Three sample sizes were considered viz 100, 150 and ,200, and 2000 replications were done in each case. Results for the bias, variance and the mean square error of prediction were computed for the four methods, though Tables 1 to 4 report results only for the Hannan Inefficient method and what we have called as the modified Hannan method (for the sample size 150).

The overall results of our simulation exercise point to the following conclusions

(1) For both models 1 and 2 the modified procedure significantly outperforms Hannan Inefficient procedure in terms of bias, variance and the mean square error of prediction . The difference is more pronounced for model 2 rather than model 1 indicating that the relative advantages of the modified procedure become more effective as the lags in the model increase.

(2) So far as comparison with the OLS methods is concerned if the lag is correctly specified then OLS estimates outperform both the original and the modified Hannan's procedures. However if lags are misspecified then the OlS estimates can be substantially off the mark and their performance can be

well below that of our modified procedure and often below that of Hannan's original procedure. The latter conclusion differs from that of Cargill and Meyer (1974).

(3) AB in the Carqil1-Meyer study, the Almon laq procedure performs poorly.

(4) Finally the overall conclusions remain robust across the three sample sizes we have considered.

While simulation studies can never furnish conclusive evidence, the general outcome of our exercise may be viewed as providing favourable evidence for our method with moderately sized samples*

6. NON-STATIONARYCASF

Suppose now that we drop the assumption of stationarity of tY(t)) and {X(tH and also allow the distributed laq coefficients b(X) to vary with time. Then analoqously to (2) we have the relation

$$Y(t) = \int_{-\infty}^{\infty} b_t(\lambda) X(t-\lambda) d\lambda + e(t) \tag{33}$$

To analyse the problem of determining $b_t(\lambda)$ we need to resort, to the concept of a time-varyinq spectrum. Even though several alternative concepts for a time-varying spectrum have been suggested in the literature, the "evolutionary spectrum" introduced by Priestley(1965> and fully elaborated in Priestley (1900),has enjoyed special appeal both because it carries a cognisable physical interpretationand because it encorapases several of the other approaches aa special cases. It is defined for a special class of processes theso-called "oscillatory" processes which are nevertheless general enough to include many processes occurring in the real world.(The definitions of evolutionary spectrum and oscillatory processes may be recovered from Priestly(1900)).

For a given set of T observationson }X(t)} and {Y(t>}, the evolutionary spectra $f_{t.xx}(w)$ and $f_{t.vv}(w)$ may be obtained by using the "double window" technique suggested by Priestley (1988) (see Nachane and Ray for an economic application). The evolutionary cross spectrum $f_{t.vx}(w)$ can also be defined by analogy with the stationary case (see Subha Rao and Tong(1972).

Suppose now that for fixed t, we define

$$H_t(w) = \int^{\infty} b_t(\lambda)\exp(iw\lambda)d\lambda$$

In an important contribution Tonq(1971) demonstrated that

$$f_{t,yx}(w) \cong H_t(w)\ f_{t,xx}(w) \tag{35}$$

This ofcourse, means the H, (w)can be regarded as the time varying generalisation of the transfer function H(w) considered earlier. Subbaa Rao and Tong (1972) now define the time varying estimators $\hat{b}_t(\lambda)$ as inverse Fourier transforms of $\hat{H}_t(w)$ for different t.

$$\hat{b}_t(\lambda) = \alpha\int_{-\infty}^{\infty} \exp(iw\lambda)\ \hat{H}_t(w)\ dw$$

Our estimators for the causal case can now be derived virtually paralleiy from the analysis of Section 2 for each fixed t (under the same assumptions made therein).

The discrete case can be handled similarly and without too much difficulty . The distribution of such time varying parameters however has proved difficult to derive even for the two-aided case, leave alone the one- sided case of interest to us.

7.CONCLUSIONS

In this paper we have suggested a class of estimators for the distributed lag problem which are modifiations of those suggested by Hannan(1963). Hannan's Inefficient estimators • are important from the applied point of view since they impose virtually no restrictions either on the lag length or the lag structure. However their major limitation is that they postulate a two-sided distributed lag system which is physically unrealisable.

Our method tries to eliminate the above limitation by considering a one-sided or causal system. We present methods to determine estimates in this case for both the continuous and the discrete distributed lag systems. The distribution of the estimates in the latter case turns out to be normal. Our simulation exercise provides considerable support in favour of the new method. Lastly we also show how this method can b generalised to the time-varying case but no distributonal results can be presented for this case as of now.

We have exclusively focussed on the situation where both input and output series are scalar series. The generalisation to the vector case presents no special difficulty -our only **reason** for not presenting the vectorial case here was to Maintain clarity of exposition.The next logical question to pose is **whether** the Hannan Efficient Estimates can be similarly **modified. The** answer is in the affirmative but the distributional considerations relating to the estimates are expected to be formidable.

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