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ON SOME UNPLEASANT MONETARIST ARITHMETIC

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***Abstract:* By formalizing the steady-state linkages between money, inflation, deficit and debt, we analytically establish the Sargent-Wallace (SW) result mat, under certain conditions, the long-run inflation rate is inversely related to monetary accommodation Thus the paper improves upon the original SW version which had to resort to numerical simulations to prove this point We also show that incorporating the Mundeil-Tobin and Darby-Tanzi effects into the model indicates further conditions that would (in)validate the SW result*.**

ON SOME UNPLEASANT MONETARIST ARITHMETIC

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By formalizing the steady-state linkages between money, inflation, interest, deficit and debt, we analytically establish the Sargent-Wallace (SW) result that, under certain conditions, the long-run inflation rate is inversely related to monetary accommodation. Thus the. paper improves upon the original SW version which had to resort to numerical simulations to prove this point. We also show that incorporating the Mundell-Tobin and Darby-Tanzi effects into the model indicates further conditions that would (in)validate the SW results.

1. Introduction

As this paper is a commentary on "Some Unpleasant Monetarist Arithmetic" by Thomas Sargent and Neil Wallace (1981), we initiate the discussion by providing a brief restatement of the Sargent-Wallace (SW) results.

In their paper, SW consider two simple macroeconomic models. The first consists of two equations, one being the government budget constraint given by [see SW eq. (1)] :

$$BD = M + D \qquad (l.l)$$

where BD is the budget deficit (net of interest payments), M is the monetary base, and D is the stock of privately held government debt. The second equation of the SW Model I is the simplest version of the quantity theory [see SW eq. (5)], i.e.,

$$P - Mv/y \qquad (1.2)$$

where P is the price level, v is the (assumed constant) velocity of money, and y is real GNP. In the SW Model 2, this equation is replaced by the money demand function [see SW eq. (10)] :

$$M/(Py) = \alpha - \beta\pi \quad (1.3)$$

where π is the rate of inflation.

In the case of their first model, a reasonable translation of the SW results yields (SW Result 1) : If the instantaneous real rate of interest is a constant r, output is growing exogenously at a given rate g, and the steady-state debt-income ratio is constant, then it must be true that high deficits lead to high inflation. Proof: By hypothesis, in steady-state, the growth rate of debt *(6)* equals the inflation rate plus the real growth rate, i.e., $\delta = \pi + g$. By the quantity theory, eq. (1.2), it must also be true that $\mu = \pi + g$, where μ is the growth rate of money. It thus follows that $\delta = \mu$ and $\pi = \delta - g$ in steady-state and, therefore, if large deficits cause *6* to be high, then μ and π must also be large in steady-state.

In the case of their second model, a reasonable translation of the SW results is only slightly more difficult. The first part of the translation would read (SW Result *2)* : Given a constant exogenous real rate of interest r which is greater than the exogenous natural rate of growth g, a constant debt-income ratio in Steady-state, and some regularity conditions that resolve problems of nonexistence and uniqueness, then one pan determine the steady-state value of inflation. Sketch of; proof: Mimicking the solution strategy in SW s Appendix B, we obtain the following nonlinear differential equation in inflation [see SW eq. (B5)]:

$$\dot{\pi} = \beta^{-1}[-f - (r-g)d + \alpha g + (\alpha-\beta g)\pi - \beta\pi^2] \quad (1.41)$$

where f and d represent constant steady-state deficit-income and debt-income ratios, respectively. One can then derive a steady-state value of π that will be the smallest possible sustainable value in steady-state. Denote this value by $\pi^*(d)$.

The final claim of the SW paper makes use of the second model (SW Result 3): Consider a situation whereby the initial stock of debt and money supply is given, and the time path of deficits (net of interest) is fixed and positive for time between 0 and T, and fixed at zero for all time beyond T. Then a low initial path for money supply (i.e., for M(t), $0 \leq t \leq T$), £an lead to a higher value for $\pi^*[D(T)/P(T)y(T)]$ than a higher initial path for money supply. Proof: The proof of this result consists only of numerical examples (described on pp. 5-6 of SW).

2. The Restatement

Despite the widespread interest that this paper has generated in the literature, the fact remains that the basic SW result - that the long-run steady-state inflation rate is inversely related to the extent of monetary accommodation - has only been proved numerically. In order to prove the SW results analytically, we need to formalize the dynamic long-run steady-state nature of the linkages between money growth, inflation, the interest rate, deficit and debt.

We base our model upon the simplest version of the quantity theory, i.e., eq. (1.2), from which we obtain the following long run relationship between the rate of inflation (π), money growth (M) and real output growth rate (g):

$$\pi = \mu - g \tag{2.1}$$

under the assumption that velocity (v) is constant.

We then invoke the Fisher equation which specifies that the nature of the relationship between the nominal interest rate (i) the real rate of interest (r) and the inflation rate is given by:

$$i = r + \pi \qquad (2.2)$$

Thus, assuming r to be a constant implies that an increase in inflation is fully reflected in nominal interest rates (The result of the assumption that $\delta r/\delta \pi \neq 0$ will be examined later) .

The budget deficit (BD) is equal to the sum of: (1) the primary, or non-interest, deficit which we can write as the primary budget share in nominal GNP (x) times nominal GNP (Py) , i.e., xPy, and (2) interest payment on the public debt which is equal to the nominal interest rate (I) times the stock of outstanding debt (D) , i.e., iD. Therefore, we have:

$$BD = xPy + iD \qquad (2.3)$$

Dividing the above expression by nominal GNP (Py) yields:

$$f = x + id \qquad (2.4)$$

where f is the deficit-income ratio (BD/Py) and d is the debt-income ratio (D/Py) .

Given the government budget constraint, eq. (1.1), and writing 6 for the proportion of the budget deficit accommodated by the monetary authorities, i.e., the proportion of the deficit covered by addition to the money stock, we have:

$$M = \theta BD, \qquad 0 \leq \theta \leq 1$$

(2.5)

Dividing eq. (2.5) by Py; rewriting M/Py as the product of (M/M) and (M/Py) ; and then invoking eq. (1.2) to replace M/Py by the inverse of velocity; yields the following solution for the rate of money growth ($\mu = \dot{M}/M$) :

$$\mu = \theta v f \qquad (2.6)$$

From eqs. (1.1) and (2.5), we have:

$$D = (l-\theta)BD \tag{2.7}$$

Dividing eq. (2.7) by Py; rewriting D/Py as the product of (D/D) and (D/Py); and then invoking the condition that, in steady-state, the growth rate of debt equals the inflation rate plus the real growth rate, i.e., D/D = $\pi+g$; yields the following solution for the steady-state debt-income ratio (d = D/Py):

$$d - (l-6)f/(\pi+g) \tag{2.8}$$

Eqs. (2.1), (2.2), (2.4), (2.6) and (2.8) constitute the model defining inflation, interest, deficit, money and debt. In order to establish the SW contention, we have to obtain an analytical solution for,the inflation rate (IT) and prove that if r > g, then the long-run steady-state rate of inflation is inversely related to the extent of monetary accommodation (G). 3. The Proof

Using the quantity theory, eq. (2.1), which states that, in steady-state, $\mu = \pi + g$, we initially replace $\pi+g$ in eq. (2.8) by μ and then use the money growth formulation, eq. (2.6), to replace μ by its definition. Doing so yields:

$$d = (l-e)/6v \tag{3.1}$$

It is thus seen that if 6=1, i.e., complete monetary accommodation, then d=0; if, on the other hand, 6=0, i.e., no accommodation whatsoever, then $d=\infty$, i.e., the debt-income ratio would explode without limit. Any intermediate value of 6 would yield a positive value for the steady-state debt-income ratio.

Substituting eqs. (2.6) and (2.4) into eq. (2.1) yields:

$$\pi - 9v(x + id) - g \tag{3.2}$$

Substituting eqs. (2.2) and (3.1) into eq. (3.2) yields:

$$\pi = \theta v[x + (r+\pi)\frac{(1-\theta)}{\theta v}] - g \qquad (3.3)$$

Solving eq. (4.3) above in terms of π yields the following expression for the long-run steady-state rate of inflation:

$$\pi = \frac{(r-g)}{\theta} + (vx-r) \qquad (3.4)$$

The result indicates that if r > g, long-run inflation can be minimized only by setting 0= 1 . This would yield π = vx-g.

Thus, we analytically establish the SW contention that if the constant exogenous real rate of interest is greater than the exogenous natural rate of growth, then a higher (lower) value of the monetary accommodation coefficient will lead (and not "can lead" as implied by the SW numerical simulations) to a lower (higher) rate of long-run steady-state inflation.

NOTES

1. The assumption that r > g implies that constancy of the real interest rate is critical in the SW model. However, analysis of the effect of a change in inflation on the real rate of interest along the lines of the Mundell-Tobin "real balance effect" (Mundell 1963, Tobin 1965) and the Darby-Tanzi "tax adjusted Fisher effect" (Darby 1975, Tanzi 1976) have indicated that the Fisher equation, eq. (2.2), should actually be modified to:

$$i = r + \beta\pi \qquad \text{(N.I)}$$

where $\beta \neq 1$. In the case of the Mundell-Tobin effect, B< 1 implying that $\delta r/\delta\pi < 0$; while in the case of the Darby-Tanzi effect, B > 1 implying that *6r/6ir* > 0. Therefore, in both cases, the assumption of a constant real rate of interest is violated.

In order to examine the implications of such a violation on the SW results, we incorporate eq. (N.I) into the model, instead of the original Fisher equation. This yields the following modified expression for the long-run steady-state inflation rate:

7r - (r - g + Gvx - 9r)/(l - B + B0) (N.2)

Setting $\delta\pi/\delta\theta < 0$ yields the following restriction on *&* for the SW conjecture to be valid under the present circumstances:

B > (vx-r)/(vx-g) (N.3)

Two very important results emerge from the above equation: (1) Even if r > g, the SW results can still fail to hold if B does not satisfy eq. (N.3) above, and (2) Even if r < g, the SW results can still hold provided S satisfies eq. (N.3) above. The following two numerical examples will illustrate this point:

Case *I (r> g) :* Let r = 0.05; g = 0.03; x = 0.03; and v = 3. From eq. (N.3), it is seen that B < 0.67 for the SW results to be invalid. Thus, let B = 0.6. Now, if 0 = 0.5 from eq. (N.2) it is seen that π = 0.0571; while if 8 = 1, π = 0.06, thereby disproving the SW results. The fact that the SW contention is not established despite r > g indicates that what really matters in the SW framework is not the ex-ante real rate of interest (r) but the ex-post real rate in steady-state $(r^*=i-\pi=r+\beta\pi-\pi)$. Thus,while the ex-ante real rate is more the real growth rate, the ex-post real rate in steady-state (which works out to be 0.0272 at 0=0.5; and 0.0260 at 0=1) is seen to be less than the real growth rate, which is why the SW results are not validated.

Case 2 (r < g) : Let r = 0.03; g = 0.05; x = 0.03; and v = 3. From eq. (N.3), it is seen that B > 1.5 for the SW results to be valid. Thus, let £ - 1.6. Now, if 0 = *0.5,* from eq. (N.2) it is seen that 7r = 0.05; while if 6 = 1, π =0.04, thereby proving the SW results. Once again, the fact that the SW contention is established despite r < g indicates that what really matters in the SW framework is not the ex-ante real rate of interest but the ex-post real rate in steady-state. Thus, while the ex-ante real rate is less the real growth rate, the ex-post real rate in steady-state (which works out to be 0.06 at 9=%; and 0,054 at 8=1) is seen to be higher than the real growth rate, which is why the SW results are not contradicted.

Thus, it is seen that even: (1) if r > g, the SW results will not pass through if *6r/6ir* (= S-l) is small enough as a result of the Mundell-Tobin effect to yield r*= r + $(\delta r/\delta\pi)$*it* < g in steady-state, and (2) if r < g, the SW results will still pass through provided $\delta r/\delta\pi$ is sufficiently large enough as a result of the Darby-Tanzi effect to ensure that r* > g in steady-state.

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