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MANAGER, WORKER, RENTIER AND GOVERNMENT: MARKOV AND NON-MARKOVIAN PLAY

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Abstract

We decompose the representative agent into her manager, worker and rentier selves. The criterion is the information set of each of the 'incarnations' at every stage of a repeated game. The rentier's information set includes anticipating information while the information set of the other agents is feedback. The equilibrium that results is, therefore, not subgame perfect. Enter the government that produces a good that complements the existing good. It is financed by the issue of money that substitutes for the asset. The enlargement of the state space of the game induces cooperation between the agents.

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1. INTRODUCTION

The aspect of the general equilibrium research programme that has troubled its astute proponents the most is the use of the device of the representative agent to explain macroeconomic outcomes. Work has begun, at the highest levels, to embrace agent heterogeneity on the basis of the distribution of characteristics *across agents.* The representative agent, then, would be the special case of the distribution collapsing to a point. The assumptions about the distribution of preferences that would generate the dispersion of behavior have not yet been specified (Kirman, 2001).

The present exercise diverts least from the standard model and might be described as a consideration of the distribution of characteristics *across the representative agent.* Specifically, the dispersion of the various 'selves' is along the time dimension. Recourse is taken to the notion of "multiple selves" incorporated into economics, among others, by Thomas Schelling, a venerable iconoclast within the game theory camp (Schmidt, 2002). Agents, in this view, are not one-dimensional with identifiable tastes and memories. Cognitive abilities are not smoothly continuous over time. It is possible that there is a predilection to switch identities, from manager to rentier to worker and back, not discontinuously but with sufficient oscillation to impact on decisions that entail binding long-term commitments as against ephemeral contacts. The discount rate is one dimension along which the selves can be distinguished. The player

who is only concerned with the moment would be one who had an instantaneous discount rate of one hundred percent. The particular strand of the multiple selves literature developed here is concerned with sustaining cooperation in social dilemmas over time (Frederick, Loewenstem and O'Donoghue, 2002). The outcome requires the cooperation of a series of "temporally situated selves". The defection of one self entails the unraveling of the equilibrium because other selves follow suit.

Otherwise, the assumptions are standard. Information is conveyed by the price vector that plays a coordinating role. At each point of time, managers, workers, rentiers, observe (relevant components of) the price vector and make bids and offers. These strategies might replace those that were made in the previous period. Mutually beneficial trades take place and reveal information through the value of the state vector and so on in repeated play. The next section elaborates on a state-space representation of the dynamic general equilibrium model. The solution is shown to be subgame imperfect.

The entry point of the government would naturally be here. There is a common perception about the precise delineation of the role of the state in economic activity today. On the one hand, in many spheres less intervention is regarded as better than more. For example, because of the proclivity of central banks to abuse their monopoly of note issue, the discretion of the monetary authorities in this regard is sought to be minimised through various mechanisms. The notion of the authorities maximising a welfare function that is different from the description of a representative agent is regarded with suspicion. On the other hand, it is also held that the commodities and services that enter the utility functions of agents are only able to deliver their measure of utils if consumed jointly with commodities/services like health, education and

infrastructure, the province of provision by the state. This expectation of the state is not new. What is novel now is that these goods are to be provided by governments that meet market standards of profitability. Distortions in prices and costs in order to provide the aforementioned goods at subsidised rates will not do because of the burdensome legacy that will be bequeathed to subsequent generations thereby. An early case for removing discretion from the hands of the monetary authorities was made, expectedly, by Milton Friedman (1948). He argued for a rule according to which the rate of monetary expansion varied one-to-one with the budget deficit or surplus. The quantity of money is increased when the government has a deficit and only by the amount of the deficit and vice versa. Money is endogenous. It was, according to this scheme, an automatic stabilizer as against, at the other extreme, a system where budget deficits and surpluses were associated with changes in the stock of government bonds held by the public. A regime where the rate of monetary expansion is constant and state-independent (a formula that he was later associated with) is dynamically unstable, he argued. He therefore advocated the elimination of the possibility of private creation of money. A one hundred percent reserve system would accomplish this objective, separating the depository from the lending function of the banking system. The power to indulge in open market operations would not exist.

The only active organ of the government would be the fiscal arm. The instruments should conform to the preferences of the people. By these operations there would be no conflict between expenditure and taxation policy and the objectives of private agents Friedman cites the reduction in inequality as a community welfare goal that fiscal policy might affect. The dilemma is to combine fiscal activism with cost-

benefit analysis. It is imperative to provide health and other social expenditures without impairing the initiatives to work and save (Schwartz and Ter-Pinassian, 2000). Particular attention is given here to government programmes like low-cost housing -that, by reducing market imperfections, have positive distributional consequences. The welfare gains from such interventions are a multiple of those, such as direct income transfers, aimed at achieving a more equitable distribution directly. There need be no conflict between redistributive fiscal policy and efficiency. Inadequate nutrition and education are binding constraints on work efforts. A superior disbursement on these counts would enhance labor productivity. The government enters in Section 3, as a coordinated and coordinating monetary and fiscal authority. Its balanced budget policy is shown to coordinate the activities of the private agents. A final section concludes.

2. "MULTIPLE SELVES" IN GENERAL EQUILIBRIUM

The familiar account begins with the aggregate production function, $Y_t = F(N_t, K_t)$, that maps inputs onto the maximum feasible level of output, Y_t. *Nt is* labour employed, K_t is capital employed. Shares in the firm are traded in a stock market. The perfectly competitive firm is a price-taker and, at the beginning of period *t,* chooses output to maximise nominal cash flows given by

$$\pi_t = p_t^y F(N_t, K_t) - w_t N_t - p_t^I I_t,$$

where p_t^p, w_t and p_t^I are the prices of the private good, of labor and investment respectively and form the components of the state vector $p_t = (p_t^p, w_t, p_t^I)$. Ignoring physical deterioration of the capital stock,

$$K_{t+1} - K_t = I_t.$$

The problem of the consumer mirrors that of the firm. The demands for labor and investment represented by Shephard's lemma are matched by the supply of those inputs as the outcome of a utility-maximising exercise with respect to labor and investment respectively. Let $V(w_t; p_t^p; p_t^I)$ denote the indirect utility function of the worker where the function is increasing in the first argument. In the case of the rentier, the optimisation sub program in period *t* includes the price of the share in the next period through the capital gains component. Depict then, the indirect utility function of the rentier by $V(p_t^I, p_{t+1}^I; w_t; p_t^p)$, where the function is increasing in the first two arguments. The profit function of the manager, $\pi(p_t^p; w_t; p_t^I)$, has already been introduced. It should be noted that the multiple utility functions are different 'incarnations' of the same agent. The ordering of the components in the domains of the functions is changed so that the first component denotes the components of the state vector of defining interest to the agent under consideration.

Consider a differential game over the bounded time interval [0,7j or the unbounded time interval $[0, \infty]$. One can be reduced to the other (Dockner, Jorgensen,

Van Long and Sorger, 2000). The state of the game at each instant $t \in [0,T]$ is described by a vector $p_t \in P$, where $P \subseteq R^3$ is the state space of the game. The initial state of the game is given by $p_0 \in P$. Players 1, 2 and 3 are the manager, the worker and the rentier respectively. At each stage $t \in [0,T]$ of the game, each of them chooses a quantity variable $x_t^i (Y_t, N_t,$ and I_t respectively),from $i \in \{1,2,3\}$, her set of feasible controls $X^i \subseteq R$. The state of the game evolves according to the differential equation

$$\dot{p}_t = f(p_t, x_t^1, x_t^2, x_t^3) \text{ with given } p_0 \in P.$$

Denote by *X* the set of all triples $x = (x^1, x^2, x^3)$ with the property that $x^i : [0,T] \to R$ is a measurable function for all $i \in \{1,2,3$ and there exists a unique, absolutely continuous state trajectory /^such that the system dynamics and the feasibility constraints given above are satisfied. An information structure of the game is a mapping

$$I : \{ (x,t) \mid x \in X, t \in [0,T] \} \to Y^1 \times Y^2 \times Y^3,$$

where Y^i is the observation space of player *i.* In the case of the first two players, the information structure is Markov. That is, for i = 1,2,

$$I^i(x,t) = (p_t^p; w_t; p_t^I),$$

whereas the rentier's information set is anticipating:

$$I^3(x,t) = (p_t^p; w_t; p_t^I, p_{t+1}^I).$$

Each player $i \in \{1,2,3$ seeks to maximise her objective functional

$$J^i(x^i) = \int_0^T e^{-r^i t} U^i(p_t, x_t^1, x_t^2, x_t^3) dt.$$

Here *U* is player i's utility function and *r* is her rate of time preference. In the game under consideration we work with indirect utility functions given by V^i, where $V^1 \equiv \pi$ and $V^2 \equiv V \equiv V^3$. The game is denoted by $\Gamma(p_0)$.

A Nash equilibrium (NE)is a triple of strategies (x^{*1}, x^{*2}, x^{*3}) such that given the equilibrium strategies of the others, no player has an incentive to deviate from her own strategy. In the case under consideration, the rentier's information set is based on the expected value of the state variable. In period *t.* +1, it is in her interests to depart from

the action dictated by the initial strategy and employ a different action if it is in her interests to do so. Her payoff thereby increases and, in the case of interest, the payoffs to the others fall, the benchmark for comparison being the NE. The perfect NE is. stronger than the time-cons is tent NE. The optimal Nash strategies should represent optimal behavior not only along the equilibrium state trajectory but also off the trajectory. Formally, the NE is subgame perfect if, for each $p_t \in P \times [0, T]$, the subgame $\Gamma(p_t)$ admits a NE $(\tilde{x}^1, \tilde{x}^2, \tilde{x}^3)$ such that $\tilde{x}^i(p_t) = x^{*i}(p_t)$ for all $i \in \{1,2,3$ and all $p_t \in P \times [t, T]$. In the present instance, we have the following:

PROPOSITION 1. The Nash equilibrium in the game T(*P*o) *[s] subgame imperfect.

Proof. Let (x^{*1}, x^{*2}, x^{*3}) be the NE of the game and the $p^*_{(.)}$ state trajectory corresponding to those strategies. At time *t,* the information set of the rentier is given by $(p_t^{*p}; w_t^*; p_t^{*I}, p_{t+1}^{*I})$.

When period *t* +1 actually arrives, an incentive to deviate from the action dictated by the strategy above must be that the price of the share is greater than expected in period /. It would be the case that

$$V(., \tilde{p}^I_{t+1}; w^*_{t+1}; p^{*p}_{t+1}) \geq V(., p^{*I}_{t+1}; w^*_{t+1}; p^{*p}_{t+1}),$$

for some $\tilde{p}^{I}_{t+1}$ that exceeds the expected p^{*I}_{t+1} by some multiple $\alpha > 0$, say. Indeed, for

a dynamical system that delivers a state vector $\tilde{p}_{t+1} \geq p^{*}_{t+1}$, where the former is greater

than the latter by the factor α, the worker is not worse off. That is,

$$V(\tilde{w}_{t+1}; \tilde{p}^{p}_{t+1}; \tilde{p}^{I}_{t+1}) = V(w^{*}_{t+1}; p^{*p}_{t+1}; p^{*I}_{t+1}).$$

The worker's indirect utility, recall, is homogenous of degree zero in the price of the product and income. In a similar manner, since the profit function is homogeneous of degree one in output and input prices,

$$\pi(\tilde{p}^{p}_{t+1}; \tilde{w}_{t+1}; \tilde{p}^{I}_{t+1}) \geq \pi(p^{*}_{t+1}; w^{*}_{t+1}; p^{*}_{t+1}).$$

All players are (weakly) better off reneging on their earlier commitment with a NE given by $(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ at stage *t+1.*

It turns out that the objective functions of the multiple selves are monotonic under the sorting condition. That is, a higher value of the state vector leads to a change in strategy and an increase in utility. The result is not different from disequilibrium accounts of the tatonnement process. Observed prices are, typically, not prices in the standard sense. However, transactions are conducted at these prices and are

information for all agents. The outcome is price dynamics that may not converge to the equilibrium state.

3. MONEY IN THE BUDGET CONSTRAINT

The role of a mediator resides precisely here (Schelling, 1960). A mediator is an agent who does no more than facilitate an efficient outcome by effecting communication between the players. In that capacity, a mediator imposes bounds on actions but, more importantly, can enlarge the space within which the play takes place. By her own strategies, a mediator compels the other players to recognise a focal point. Rational players will be motivated to set aside certain channels of communication and block off routes to memory and anticipating information.

The role of mediator in our model is played by the government, which is a composite fiscal cum monetary authority, and which provides a public good X^4 which complements the private good consumed by the agents. An increase in the consumption of the good, consumption of the private good being held constant, results in an increase in utility. The good is financed by a profit-maximising government by the issue of its liability, money, at a price p^m. The institution of interest-bearing fiat money, many believe, is almost with us (Goodfriend, 2002). The environment is one in which central banks the world over have moved from price and quantity controls to market-based procedures for conducting monetary policy. Furthermore, the general trends depict a reduction in reserve demand ostensibly due to the absence of interest on reserves. If, in the future, banking systems developed an electronic system for the settlement of accounts

independent of central banks and currency is displaced by electronic devices, the monetary base would lose its medium-of-exchange role. If, on the other hand, central banks moved to paying interest on reserves, their leverage on interest rates would remain secure. Banks could exchange currency for reserve balances at the policy rate which would equal the market rate of interest. They would regard deposits at the central bank as government debt with a variable rate of interest determined continuously by market forces.

The indirect utility function of the consumer is decreasing in the price of money. Money and bonds are substitutes. The indirect utility function of the rentier is, therefore, increasing in the price of this new asset. The information in the form of complements and substitutes enlarges the state space of the game. The information that any player can now obtain about the strategies of the others and which she, in turn, can convey about her own strategy, modifies the data of the game according to the specific shape of the players' preferences. The solution to the coordination problem of game $\Gamma(p_0)$ can be Schelling's focal point which is common to all players and which serves as an anchor for their anticipations. Government expenditure and money are a common basis for their mutual expectations. The role of the state is the environment in which all I players act without being part of the game. Monetary and fiscal policy, in the sense introduced, fulfill the criteria that focal points are expected to meet like 1. the asymmetric role played by the government, 2. policies that are nontrivial and that can be interpreted I in an unambiguous way and 3. the choice of strategies that change behavior in I accordance with the prescriptions deriving from such a common interpretation (Schmidt,

2002). The state of the game in period *t* is now represented by $p_t = (p_t^p; w_t; p_t^l; p^m)$. Denote the game by $\Gamma'(p_0)$.

PROPOSITION 2. The Nash equilibrium in the game $\Gamma'(p_0)$ is subgame perfect.

Proof. Assume, as before, that $(x^{*1}, x^{*2}, x^{*3}, x^{*4})$ is the NE of the game and that $p^*_{(.)}$ is the state trajectory corresponding to these strategies. The observation set of the individual as rentier at time *t* is $(p_t^{*p}; w_t^*; p_t^{*l}, p_{t+1}^{*l}; p_t^{*m}, p_{t+1}^{*m})$. Note that the future price of money enters the information set of the rentier along with the future price of its substitute asset by the fundamental principle of valuation. Let the incentive to deviate from the prior commitment strategy at time *t* be that the actual price of money $\tilde{p}_{t+1}^m \geq p_{t+1}^{*m}$. Then,

$$V(.,p_{t+1}^{*l}; w_{t+1}^*; p_{t+1}^{*p};.., \tilde{p}_{t+1}^m) \geq V(.,p_{t+1}^{*l}; w_{t+1}^*; p_{t+1}^{*p};.., p_{t+1}^{*m}).$$

However, as a worker-consumer, for any $\tilde{p}_{t+1}^m \geq p_{t+1}^{*m}$,

$$V(w_{t+1}^*; p_{t+1}^{*p};.., p_{t+1}^{*l};.., \tilde{p}_{t+1}^m) \leq V(w_{t+1}^*; p_{t+1}^{*p};.., p_{t+1}^{*l};.., p_{t+1}^{*m}),$$

a contradiction.

Reference might be made here to the work of Jean-Pascal Benassy (2002) who, in a non-Walrasian framework, develops models of maximizing governments that undertake policy on the basis of information sets that are never more refined than those of the private sector. In every period, private sector agents and the monetary cum fiscal authorities are free to act simultaneously. He demonstrates that the optimal policies are activist.

4. CONCLUSION

According to one line of research, the sine qua non of Keynesian economics is the coordination problem. The coordination game is the basis of the stability of institutions and the phenomenon of leadership (Schelling, 1960). Among the many candidates for equilibria, the government suggests a particular behavioural rule that everyone can expect everyone else to be conscious of as a conspicuous candidate. Others cannot be identified by means of bilateral or multilateral bargaining. The institution of capitalism is founded on managers, workers and rentiers playing their assigned roles in the mode of production. The government, in its role as issuer of legal tender and provider of social infrastructure, is a necessary adjunct intermediating between the groups. It hastens the "convergence of expectations".

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