University of Mumbai

DEPARTMENT OF ECONOMICS

Berge-Bacharach-Schelling Play

by

Romar Correa

WORKING PAPER UDE 38/1/2012

JANUARY 2012

ISSN 2230-8334

DEPARTMENT OF ECONOMICS
UNIVERSITY OF MUMBAI
Vidyanagari, Mumbai 400 098

Documentation Sheet

Title:

Berge-Bacharach-Schelling Play

Author:
Romar Correa

External Participation:

WP. No.: UDE 38/1/2012
Date of Issue: January 2012

Contents: 10 p, -- T, 5 F, 15 R
No. of Copies: 100

Abstract

We play coordination games. The Nash equilibrium is not usually the cooperative solution. According to the definition, the choice variable is the strategy of a player given the strategies of all the other players. The Berge equilibrium provides the cooperative outcome in many coordination contexts. Now, the choice vector is the vector of strategies of all the players, given a player's strategy. We explore the Helly metric that underlies both solution concepts. The metric does not satisfy the 'separation of points' criterion. Michael Bacharach and Thomas Schelling suggest that people do distinguish between strategies even when the payoffs are the same. We take the step from Nash-Berge to Bacharach-Schelling by specifying a metric. The metric gives a leader who chooses an outcome for every strategy pair.
.

Keywords: indeterminacy; leadership

JEL Code(s): B31; C60; C72

Berge-Bacharach-Schelling Play*

1 Introduction

We are interested in cooperation in social dilemmas and the related phenomenon of coordination in common interest games. The Nash Equilibrium (NE), in general, does not give the highest aggregate payoffs. Yet, people cooperate in prisoner's dilemmas, chicken and trust games to their joint benefit. Each player makes the utility of others a key feature in her reasoning if she believes that others think analogously. Consequently, over the last few years, great expectations have been set on the concept of Berge Equilibrium (BE) as providing an approach to the problem of cooperation in noncooperative settings (for some treatments see Coleman, Körner, Musy and Tazdaït, 2011; Courtois, Nessah and Tazdaït, 2011; Deghdak, and Florenzano, 2011).

According to the definition, while the optimization exercise is still individualistic, agent i maximizes her utility over the strategies of all the other players j given her strategy in the joint plan. Note the similarity with the definition of the NE: Agent i maximizes her utility over her strategy set taking as given the strategies of all the other players j in the NE. Why should player i want to maximize the utility of the other players j? Edna Ullman-Margalit offers a minimal requirement in "considerateness" defined as actions designed to decrease the discomfort of others at trivial costs to oneself (Ullman-Margalit, 2011). The 'other' could be anonymous as well as familiar. Gratitude may or may not result but reciprocity is expected. Considerateness can be distinguished from other well-known notions. Thus, 'altruism' is the case when the costs and benefits to others are significant. Considerateness is the minimal acknowledgement of the existence of the other qua respect for others as human beings and falls short of 'kindness'. The expectation is that others are considerate but, at the same time, the considerate act of the agent contributes towards fostering such an environment.

In the literature, players are free to play Nash or Berge. The indeterminacy of Nash equilibria is substituted by the indeterminacy of Nash or Berge equilibria. The two-player Prisoner's Dilemma is a special case of the n-person Prisoner's Dilemma. In both cases defection is the unique NE and cooperation is the unique BE. Both equilibrium notions

* I am grateful to the insights of a reviewer. The usual disclaimers apply.

run side by side. The NE represents the individualistic idea within interdependence theory while the BE stands for the social value orientation within that theory. In this regard, Courtois, et.al. distinguish between moral sentiments and other-regarding preferences. According to the latter, individual utility functions have two arguments, one concerned with self-welfare, the second with the welfare of others. Both Nash and Berge behavior rules can be rationalized according to this extended definition. The reference to moral sentiments leads to Adam Smith's *Theory of Moral Sentiments* from which this indeterminacy in historical time can be traced (Álavrez & Hurtado, 2012). Since the eighteenth century, individuals recognized their fellow-beings as extensions of themselves. With the onset of social Darwinism, this sympathy gave way to individual fitness leading to suboptimal competitive interactions between people. The end of the last century and the beginning of the present have witnessed a renewed interest in social interactions on foundations other than self-interest. Through sympathy, an agent places herself in the skin of other agents and experiences what she would feel if she was actually in their situations. The movement is instantaneous and triggered by specific circumstances. If the feelings coincide in the particular game, agent *i* will cooperate with agents *j*. However, if they do not, agent *i* will 'defect'. Sympathy is the basis of moral judgments. Both Nash and Berge norms are internalized.

The source of indeterminacy in the NE is the rational agent's attempt to achieve the best-act profile by choosing only one component of the profile. The BE makes the substitution that it is the best-act profile of the others that the individual deliberates over. What is parametric in NE is choice-theoretic in BE and vice versa. The way out according to Michael Bacharach and Thomas Schelling is to consider the agent choosing the whole profile. Between themselves, agents will implement their profile choices. If all play their part in the solution vector it will be unique. Bacharach and Schelling pioneered reasoning in one-shot engagements. The mainstream response is to repeat the game with learning in infinite interaction. Attempts to understand Bacharach's 'team reasoning' have run the gamut from philosophy to neuroscience (Al-Suwalem, 2012). Players might obey Kant's categorical imperative: Do unto others as you would have others do unto you. The effectiveness of a strategy depends on the others who adopt it. Thus, deviation in a prisoner's dilemma is inconsistent. Players deviate to gain not to lose. If deviation is universalized, all lose. In other words, the equilibrium does not satisfy Al-Suwalem's criterion of universalizability. Other criteria offered to distinguish ecological or social rationality from individual rationality include 'reversibility'. Accordingly, in two-player games actions are chosen which are optimal if each player put herself in the other's shoes. The axiom is extended to groups by iterating the rule. The 'reversible rule' thus becomes universalizable. Neuroscience contributes 'empathy'. 'Mirror neurons' enable a player to identify with others. Feelings and emotions connect spontaneously. It is more costly for the neural system to isolate feelings than to relate to interacting parties. Our brains are hardwired with cooperative mechanisms. A final precept that will be reflected below is the axiom of 'symmetry or invariance under the permutation of individuals'. Players displaying groupthink search for a profile of strategies that maximizes the collective payoffs of the group. That profile need not be a Nash equilibrium. If one payoff Pareto-dominates the others, that payoff clearly maximizes the collective payoff. If the optimizing payoff is unique, members of the team play their component strategies in the profile. If not unique the theory is indeterminate.

In addition, both Bacharach and Schelling would question the assumption that calling 'Heads' and 'Tails' is completely symmetrical. In classical game theory any additional data contained in the two labels is deleted. However, it is precisely by recognizing that 'Heads' and "Tails' carry different meanings for different individuals that coordination problems are solved.

2 On Leadership

Many have proposed that a leader must emerge from among players to bring about a determinate solution. For instance, in an experiment on a one-stage Prisoner's Dilemma game with a finite strategy space the unique cooperative outcome emerges when players alternative in exerting power to end the game (Attanasi et.al., 2011). Again, giving by a pair is higher and more stable when one of the two is a leader (d'Adda, 2012). A step towards that end is the notion of a mediator who operates from a contractarian perspective (Sugden, 2011). A mediator shows the individuals how they can concatenate their actions to mutual benefit. The proposition runs as follows: It is in the interest of each separately to agree to perform the act together. The plan is not a set of mutually consistent instructions sent to each player separately but is a unique recommendation addressed to the collection of individuals. A key facet of contractarian reasoning is that it favors general rules rather than particular solutions. For instance, the notion of leadership recommended below does not deliver the same leader in all situations. Each player will not benefit identically from every application of the rule considered independently. Rather, each will benefit overall from the general application of the rule.

As an illustration, using material from standard psychology tests Laurence M Ball argues that groupthink is at work at the Federal Open Markets Committee (FOMC) (Ball, 2012). There is a regular tendency for members to accept a perceived majority view rather than express their personal preferences. The reason is that they value group harmony. One route to this end is the existence of a "directive leader". Ben Bernanke was ill equipped to play this role, paradoxically, when he became Governor of the Federal Reserve. He was disinclined to question the views of others because of his "quiet", "modest", "shy" manner. Team reasoning is absent in the deliberations of the Bank of England's Monetary Policy Committee. Each member of the committee is legally enjoined to vote based on an individual preference function. There is no directive leader nor a desire to seek consensus. Five-four votes on policy decisions are not uncommon and the Governor is as likely to be on the side of the minority as the other.

3 A suitable metric

Consider the following noncooperative game *G* in normal form. The notations follow Deghdak and Florenzano (2011).

$$G = (I_i, u_i)_{i \in I})$$

where $I = \{1, \ldots, n\}$ is the set of players, X_i is the set of strategies x_i of player *i*, $X = \prod_{i=1}^{n} X_i$ is the set of situations of the game *G*, u_i is $u_i : X \rightarrow \mathbb{R}$ is the payoff function of player *i*. The individual strategy spaces are assumed to be compact and, consequently, the product space is compact. Let $I\{i\} = \{j \in I : j \neq i\}$and denote $X_{-i} = \prod_{j \in I\{i\}} X_j$ and if $x \in X, x_{-i} = (x_j)_{j \neq i} \in X_{-i}$.

We have the NE given by

Definition 1. $\bar{x} \in X$ is a NE of the game if $\forall i \in I, \forall x_i \in X_i, u_i(\bar{x}) \geq u_i(x_i, \bar{x}_{-i})$.

The Nash equilibrium is immune to unilateral deviations. Given that all players *j* play their Nash strategy, player *i* has no incentive to deviate from his Nash strategy.

Less familiar is the BE:

Definition 2. $\bar{x} \in X$ is a BE of the game if $\forall i \in I, \forall j \in I \backslash \{i\}, \forall y_{-i} \in X_{-i}$,

$$u_j(\bar{x}) \geq u_j(\bar{x}_i, y_{-i}).$$

The maximization of player *j*'s utility is carried out over the strategy sets of the other players *i*. Player *j* maximizes her utility when all the other players *i* play their Berge strategies. When the other players deviate, player *j* is worse off and she is not necessarily better off by deviating herself. However, in Berge equilibrium all players *i* maximize the utility of *j* and the others. In many games, everyone is better off. This is mutual support.

For the proof of the existence of Nash equilibrium, $u_i(x_i, x_{-i})$ must be quasi-concave in x_i for all x_{-i}. Along identical lines, for the application of fixed-point theorems to prove the existence of Berge equilibrium, $u_i(x_i, x_{-i})$ must be quasi-concave in x_{-i} for all x_i. In like manner, the assumption of upper semicontinuity in the case of Nash translates to the following in the case of Berge (Deghdak and Florenzano, 2011, p.4). For each $i \in I, \forall j \in I \backslash \{i\}$, the function u_j is upper semicontinuous on $X_i \times X_{-i}$. Pursing the logic of our case, we make the following assumption:

Assumption: We assume that the payoff functions u_i are upper semicontinuous on X.

We define the upper semicontinuity of the function u_i on Xas follows with η and B

denoting appropriate neighborhoods: $\forall \varepsilon > 0, \exists \eta = \eta(\varepsilon, x)$ such that $u_i(y) \leq u_i(x) + \varepsilon, \forall y \in B(x, \eta)$.

Underlying the Nash equilibrium is the Helly metric (Vorob'ev, 1994; Wald, 1950):

Definition 3. For any x_i' and $x_i'' \in X_i$,

$$\rho_i^H(x_i', x_i'') = \max_{j \in I} \sup_{x_{-i} \in X_{-i}} \left| u_j(x_i', x_{-i}) - u_j(x_i'', x_{-i}) \right|$$

Note that the metric does not satisfy the 'separation of points' criterion of a metric. When the use of a strategy by a player leads to the same payoff for all the players, the two strategies are to be regarded as identical. Clearly, the so-called pseudometric also underlies the Berge equilibrium. If two j-1strategy vectors lead to the same payoff for player i, they are to be regarded as identical.

The metric would be of interest if the two strategies are not 'close'. Let us, in that case, consider two strategies x' and x'' that belong to different intervals as given in our assumption/definition of upper semicontinuity. Denote the epigraph of the function $u_i: X \to \mathbb{R}$ by the subset $\mathcal{E}(u_i)$ =$\{(x, \lambda) \in X \times \mathbb{R} | u_i \leq \lambda\}$. We know that the upper semicontinuity of the function u_i on $X$is equivalent to the property that the epigraph of the function u_i is closed. In the case of strategies that are 'far enough' we have

Proposition 1. The Helly pseudometric is a metric.

Proof. Let $\{(x^n, \lambda^n)\}$be a sequence of points of $\mathcal{E}(u_i)$ that converge to (x', λ). Now, $u_i \leq \lambda$. If $\{(x^m, \lambda^m)\}$ is a sequence of points that converges to (x'', λ'), then $(x'', \lambda') \in \mathcal{E}(u_j)$ for some $u_j \leq \lambda'$. That is to say, if x' is not close to $x'', u_i \neq u_j$.

In other words, $u_j(x_i', x_{-i}) \equiv u_j(x_i'', x_{-i}) \Leftrightarrow x_i' \equiv x_i''$. Strategies that give the same payoffs are identical. However, we need to distinguish between 'Heads' and 'Tails'. We require the Bacharach Schelling (BS) metric:

Definition 4. For any $x^{'} \epsilon X$ and $x^{''} \epsilon X$,

$$\rho_i^{BS}(x', x'') = \max_{j \in I} |u_j(x') - u_j(x'')|$$

Given two strategies, the maximization operation is carried out over all players j to determine the 'leader' i. It can easily be seen that the metric over (x'', x''') given by another leader $\rho_j^{BS}(x'', x''')$, generates a metric $\rho_k^{BS}(x', x''')$ and a leader k over a strategy pair (x', x'''). In other words, the 'triangle inequality' is satisfied. The metric applies to all other strategy pairs and potential leaders. The 'separation of points' criterion is met for all players but one we call the leader. The leader is the player who chooses the outcome. She would be indifferent between the two strategies. In effect, indeterminacy is removed by transforming the problem into a unilateral optimization exercise.

The following illustrations are drawn from Coleman et. al., 2011. Consider the following game when the NE and BE coincide. P_1 will clearly prefer the northeast corner, P_2, the southwest corner. Our conjecture is that one will be indifferent between the two outcomes from the Bacharach-Schelling perspective and choose a particular payoff pair. She might be a leader who maximizes the welfare of the follower or chooses a box that maximizes her own utility. In the first natural field experiment on the dictator game where subjects are unaware that they are participating in an experiment, the outcomes turn out to be counter to the standard results (Stoop, 2012). Dictators display a large measure of pro-social behavior. In similar other experiments, subjects display an equally high amount of other-oriented behavior.

		P_2	
		C_2	D_2
P_1	C_1	2,2	4,3
	D_1	3,4	1,1

In game 2a below we have representations of the "chicken" or "Hawk-Dove" game. The BE at (C_1, C_2) meets the requirement of the BB metric and is unique. Its symmetry also meets Schelling's requirement of the salience and prominence of a focal point. The NE are (D_1,C_2) and (C_1,D_2). In game 2b, it is the unique NE at (C_1,C_2) that meets the requirement of the BB metric (and a focal point). Here, there are multiple BE at (C_1,D_2) and (D_1,C_2).

2a:

		P_2: C_2	D_2
P_1	C_1	3,3	2,4
	D_1	4,2	1,1

2b:

		P_2: C_2	D_2
P_1	C_1	3,3	4,2
	D_1	2,4	1,1

Game 3a below has a unique NE at (C_1,C_2) and a unique BE at (D_1,D_2) while the transformation 3b is the Prisoner's Dilemma with a unique NE at (D_1,D_2) and a unique BE at (C_1,C_2). In both cases, the unique BB equilibrium is the outcome (3,3).

3a:

		P_2: C_2	D_2
P_1	C_1	3,3	4,1
	D_1	1,4	2,2

3b:

		P_2: C_2	D_2
P_1	C_1	3,3	1,4
	D_1	4,1	2,2

An attractive property of the metric is given by the following result.

Proposition 2. The Bacharach-Schelling metric space is complete.

Proof. We know that $\mathbb{R}$ is complete and denote the Euclidean metric by $\rho_i^E(x', x'')$. Let $\{x_n\}$ be a sequence of elements $x_n \in X$. If x_n converges to x, then $\forall \varepsilon > 0, \exists n_0 \in \mathbb{N}$ such that $\forall n, m \geq n_0$,

$$\rho_i^{BS}(x_n, x_m) \leq \left(\sum_{i \in I}(u_i(x_n) - u_i(x_m)\right)^{\frac{1}{2}} \stackrel{\text{def}}{=} \rho_i^E(x_n, x_m) \leq \varepsilon.$$

4 Conclusion

The Nash equilibrium concept has the well-known limitation of being unable to explain cooperation in a large variety of noncooperative contexts. Strangers come together, never to meet again, and cooperate. The contribution of Berge is to direct attention to the 'others' without deflecting attention away from the individual who is the subject of analysis. While players maximize their payoff functions, the domain is the product strategy space of all the other players. We explore the Helly metric underpinning both solution concepts. Rather than distinguishing between the 'self' and the 'others', Michael Bacharach and Thomas Schelling were concerned with the space of situations. We propose a measure to capture their insights. Coordination games contain multiple equilibria. Yet, people in the ordinary business of life routinely solve them to mutual benefit. Among many avenues, they suggested that a leader is required to pick a solution. Our metric generates a leader for every strategy pair. While some players will prefer to call 'Heads' and the others 'Tails' in the flip of a coin and thereby not solve the pure coordination problem, the leader would be indifferent and suggest one arbitrarily. The others would have no reason not to acquiesce in the plan.

References

Al-Suwailem, S (2012). Complexity and Endogenous Instability. Algorithmic Social Sciences Research Unit Discussion Paper Series 3-2012/1

Álavarez A and J Hurtafo (2012). "Out of sight, out of mind": Modern economics, social interactions and Smith's sympathy. Universidad de los Andes, Facultad de Economía CEDE (2012-01)

Attanasi, G, AG Gallego, N Georgantzis and A Montesano (2011) An experiment on Prisoner's Dilemma with Confirmed Proposals, December

Bacharach, M (2006). Beyond Individual Choice. Princeton University Press, Princeton

Ball, LM (2012). Ben Bernanke and the Zero Bound. NBER Working Paper No. 17826

Coleman, AM, TW Körner, O Musy and T Tazdaït (2011). Mutual support in games: Some properties of Berge equilibria. Journal of Mathematical Psychology 55, 2: 166-175

Courtois, P and RNT Tazdaït (2011). How to play the game? Nash versus Berge behavior rules. Laboratorie Montpelliérain d'Economie Théorique et Appliquée (LAMETA), Unité Mixte de Recherche, Document de Recherche n^0 2011-05

d'Adda, G (2012) Leadership and influence: Evidence from an artefactual field experiment on local public good provision. Department of Economics, University of Zurich Working Paper No.19 January 2012

Deghdak, M and M Florenzano (2011). On the existence of Berge's strong equilibrium. Centre d'Economie de la Sorbonne Working Paper 2011-39

Schelling, T C (1960). The Strategy of Conflict. Harvard University Press, Cambridge

Stoop, J (2012) From the lab to the field: envelopes, dictators and manners. MPRA Paper No.37048

Sugden, R (2011). The behavioral economist and the social planner: to whom should behavioral welfare economics be addressed? Papers in Economics and Evolution #1121, Max Planck Institute of Economics, December 20, 2011

Ullman-Margalit, E (2011). Considerateness. The Hebrew University of Jerusalem Discussion Paper #584, July 2011

Vorob'ev N (1994). Foundations of Game Theory. Birkhäuser Verlag, Basel

Wald, A (1950). Statistical Decision Functions. John Wiley & Sons, New York