UNIVERSITY OF MUMBAI

Syllabus for: T.Y.B.Sc./T.Y.B.A.

Program: B.Sc. /B.A.

Course: Mathematics

(Credit Based Semester and Grading System with effect from the academic year 2013–2014)

Revised Syllabus in Mathematics
Credit Based Semester and Grading System
Third Year B. Sc. / B. A. 2013-2014
SEMESTER V

REAL ANALYSIS AND MULTIVARIATE CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/ Week
USMT501 UAMT501	I	Riemann Integration	2,5	3
	II	Double and Triple Integrals		
	III	Sequences and series of functions		
ALGEBRA I				
USMT502 UAMT502	I	Quotient Space and Orthogonal Transformation	2.5	3
	II	Diagonalization and Orthogonal diagonalization		
	III	Groups and subgroups		
TOPOLOGY OF METRIC SPACES I				
USMT503 UAMT503	I	Metric spaces	2.5	3
	II	Sequences		
	III	Continuity		
NUMERICAL METHODS I (ELECTIVE A)				
USMT5A4 UAMT5A4	I	Transcendental equations	2.5	3
	II	Polynomial and System of linear algebraic equations		
	III	Eigenvalue problems		
NUMBER THEORY AND ITS APPLICATIONS I (ELECTIVE B)				
USMT5B4 UAMT5B4	I	Prime numbers and congruences	2.5	3
	II	Diophantine equations and their solutions		
	III	Quadratic Reciprocity		
GRAPH THEORY AND COMBINATORICS I (ELECTIVE C)				
USMT5C4 UAMT5C4	I	Basics of Graph Theory	2.5	3
	II	Spanning Tree		
	III	Hamiltonian Graphs		
PRBABILITY AND APPLICATION TO FINANCIAL MATHEMATICS I (ELECTIVE D)				
USMT5D4 UAMT5D4	I	Probability as a Measure-Basics	2.5	3
	II	Absolutely continuous Probability measure, Random Variables		
	III	Joint Distributions and Conditional Expectation		

Course	PRACTICALS	Credits	L/Week
USMTP05 UAMTP05	Practicals based on USMT501/UAMT501 and USMT502/UAMT502	3	6
USMTP06 UAMTP06	Practicals based on USMT503/UAMT503 and USMT5A4/UAMT5A4 OR USMT5B4/UAMT5B4 USMT5C4/UAMT5C4 OR USMT5D4/UAMT5D4	3	6

SEMESTER VI

REAL ANALYSI AND MULTIVARIATE CALCULUS II				
Course Code	UNIT	TOPICS	Credits	L/ Week
USMT601 UAMT601	I	Differential Calculus	2,5	3
	II	Differentiability		
	III	Surface integrals		
ALGEBRA II				
USMT602 UAMT602	I	Normal subgroups	2.5	3
	II	Ring theory		
	III	Factorization		
TOPOLOGY OF METRIC SPACES II				
USMT603 UAMT603	I	Fourier Series	2.5	3
	II	Compactness		
	III	Connectedness		
NUMERICAL METHODS II (ELECTIVE A)				
USMT6A4 UAMT6A4	I	Interpolation	2.5	3
	II	Interpolation and Differentiation		
	III	Numerical Integration		
NUMBER THEORY AND ITS APPLICATIONS II (ELECTIVE B)				
USMT6B4 UAMT6B4	I	Continued Fractions	2.5	3
	II	Pell's equation, Units and Primes		
	III	Cryptography		
GRAPH THEORY AND COMBINATORICS II (ELECTIVE C)				
USMT6C4 UAMT6C4	I	Colouring in a graph and Chromatic number	2.5	3
	II	Flow theory		
	III	Combinatorics		
PRBABILITY AND APPLICATION TO FINANCIAL MATHEMATICS II (ELECTIVE D)				
USMT6D4 UAMT6D4	I	Limit Theorems in Probability, Financial Mathematics-Basics	2.5	3
	II	Forward and Futures Contract		
	III	Options		

Course	PRACTICALS	Credits	L/Week
USMTP07 UAMTP07	Practicals based on USMT601/UAMT601 and USMT602/UAMT602	3	6
USMTP08 UAMTP08	Practicals based on USMT603/UAMT603 and USMT6A4/UAMT6A4 OR USMT6B4/UAMT6B4 USMT6C4/UAMT6C4 OR USMT6D4/UAMT6D4	3	6

Note: 1. USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503 are compulsory courses for Semester V.

2. Candidate has to opt one Elective Course from USMT5A4/ UAMT5A4, USMT5B4/ UAMT5B4, USMT5C4/ UAMT5C4 and USMT5D4/ UAMT5D4 for Semester V.

3. USMT601/UAMT601, USMT602/UAMT602, USMT603/UAMT603 are compulsory courses for Semester VI.

2. Candidate has to opt one Elective Course from USMT6A4/ UAMT6A4, USMT6B4/ UAMT6B4, USMT6C4/ UAMT6C4 and USMT6D4/ UAMT6D4 for Semester VI.

4. Passing in theory and practical shall be separate.

Teaching Pattern:

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).
3. One assignment per course or one project.

Revised Syllabus in Mathematics
Credit Based Semester and Grading System
Third Year B. Sc. / B. A. 2013-14

Semester V

Course: Real Analysis and Multivariate Calculus I
Course Code: USMT501 / UAMT501

Unit I. Riemann Integration(15 Lectures)

(a) Uniform continuity of a real valued function on a subset of $\mathbb{R}$ (brief discussion)

(i) Definition.

(ii) A continuous function on a closed and bounded interval is uniformly continuous (only statement).

(b) Riemann Integration.

(i) Partition of a closed and bounded interval $[a, b]$, Upper sums and Lower sums of a bounded real valued function on $[a, b]$. Refinement of a partition, Definition of Riemann integrability of a function. A necessary and sufficient condition for a bounded function on $[a, b]$ to be Riemann integrable.(Riemann's Criterion)

(ii) A monotone function on $[a, b]$ is Riemann integrable.

(iii) A continuous function on $[a, b]$ is Riemann integrable.
A function with only finitely many discontinuities on $[a, b]$ is Riemann integrable.
Examples of Riemann integrable functions on $[a, b]$ which are discontinuous at all rational numbers in $[a, b]$

(c) Algebraic and order properties of Riemann integrable functions.

(i) Riemann Integrability of sums, scalar multiples and products of integrable functions. The formulae for integrals of sums and scalar multiples of Riemann integrable functions.

(ii) If $f : [a, b] \to \mathbb{R}$ is Riemann integrable and $f(x) \geq 0$ for all $x \in [a, b]$, then $\int_a^b f(x)dx \geq 0$.

(iii) If f is Riemann integrable on $[a, b]$, and $a < c < b$, then f is Riemann integrable on $[a, c]$ and $[c, b]$, and $\displaystyle\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$.

(d) First and second Fundamental Theorem of Calculus.

(e) Integration by parts and change of variable formula.

(f) Mean Value Theorem for integrals.

(g) The integral as a limit of a sum, examples.

Reference for Unit I:

1. **Real Analysis** Bartle *and* Sherbet.
2. **Calculus, Vol. 2**: T. Apostol, John Wiley.

Unit II. Double and Triple Integrals (15 Lectures)

(a) The definition of the Double (respectively Triple) integral of a bounded function on a rectangle (respectively box).

(b) Fubini's theorem over rectangles.

(c) Properties of Double and Triple Integrals:

  (i) Integrability of sums, scalar multiples, products of integrable functions, and formulae for integrals of sums and scalar multiples of integrable functions.

  (ii) Domain additivity of the integrals.

  (iii) Integrability of continuous functions and functions having only finitely (countably) many discontinuities.

  (v) Double and triple integrals over bounded domains.

(d) Change of variable formula for double integral (proof for rectangular domain and invertible affine transformations) and Change of variable formula for triple integrals (no proof).

Reference for Unit II:

1. **Real Analysis** Bartle *and* Sherbet.
2. **Calculus, Vol. 2**: T. Apostol, John Wiley.
3. **Basic Multivariable Calculus**: J.E. Marsden, A.J. Tromba and A.Weinstein, Springer International Publication.
4. **A course in Multivariable Calculus and Real Analysis**: S. Ghorpade and V.Limaye, Springer International Publication.

Unit III. Sequences and series of functions (15 Lectures)

(a) Pointwise and uniform convergence of sequences and series of real-valued functions. Weierstrass M-test. Examples.

(b) Continuity of the uniform limit (resp: uniform sum) of a sequence (resp: series) of real-valued functions. The integral and the derivative of the uniform limit (resp: uniform sum) of a sequence (resp: series) of real-valued functions on a closed and bounded interval. Examples.

(c) Power series in $\mathbb{R}$. Radius of convergence. Region of convergence. Uniform convergence. Term-by-term differentiation and integration of power series. Examples.

(d) Classical functions defined by power series such as exponential, cosine and sine functions, and the basic properties of these functions.

Reference for Unit III: Methods of Real Analysis, R.R. Goldberg. Oxford and International Book House (IBH) Publishers, New Delhi.

Recommended books:

(1) Robert G. Bartle and Donald R. Sherbert, Introduction to Real Analysis, Second edition, John Wiley & Sons, INC.

(2) Richard G. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt. Ltd., New Delhi.

(3) Tom M. Apostol, Calculus Volume II, Second edition, John Wiley & Sons, New York.

(4) J. Stewart. Calculus. Third edition. Brooks/Cole Publishing Co.

(5) Berberian. Introduction to Real Analysis. Springer.

Additional Reference Books:

(1) J.E. Marsden and A.J. Tromba, Vector Calculus. Fifth Edition, http://bcs.whfreeman.com/marsdenvc5e/

(2) R. Courant and F. John, Introduction to Calculus and Analysis, Volume 2, Springer Verlag, New York.

(3) M.H. Protter and C.B. Morrey, Jr., Intermediate Calculus, Second edition, Springer Verlag, New York, 1996.

(4) D.V. Widder, Advanced Calculus, Second edition, Dover Pub., New York.

(5) Tom M. Apostol, Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.

(6) J. Stewart. Multivariable Calculus. Sixth edition. Brooks/Cole Publishing Co.

(7) George Cain and James Herod, Multivariable Calculus. E-book available at http://people.math.gatech.edu/ cain/notes/calculus.html

Course: Algebra I
Course Code: USMT502 / UAMT502

Unit I. Quotient Space and Orthogonal Transformation (15 Lectures)

Review of vector spaces over $\mathbb{R}$:

(a) Quotient spaces:

(i) For a real vector space V and a subspace W, the cosets $v + W$ and the quotient space V/W. First Isomorphism theorem of real vector spaces (Fundamental theorem of homomorphism of vector spaces.)

(ii) Dimension and basis of the quotient space V/W, when V is finite dimensional.

(b) (i) Orthogonal transformations and isometries of a real finite dimensional inner product space. Translations and reflections with respect to a hyperplane. Orthogonal matrices over $\mathbb{R}$.

(ii) Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space. Characterization of isometries as composites of orthogonal transformations and isometries.

(iii) Orthogonal transformation of $\mathbb{R}^2$. Any orthogonal transformation in $\mathbb{R}^2$ is a reflection or a rotation.

(c) Characteristic polynomial of an $n \times n$ real matrix and a linear transformation of a finite dimensional real vector space to itself. Cayley Hamilton Theorem (Proof assuming the result $A\ \mathrm{adj}(A) = I_n$ for an $n \times n$ matrix over the polynomial ring $\mathbb{R}[t]$.)

Reference for Unit I:

(1) S. Kumaresan, Linear Algebra: A Geometric Approach.

(2) M. Artin. Algebra. Prentice Hall.

(3) T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.

(4) L. Smith, Linear Algebra, Springer.

Unit II. Diagonalization and Orthogonal diagonalization (15 Lectures)

(a) Diagonalizability.

(i) Diagonalizability of an $n \times n$ real matrix and a linear transformation of a finite dimensional real vector space to itself.

Definition: Geometric multiplicity and Algebraic multiplicity of eigenvalues of an $n \times n$ real matrix and of a linear transformation.

(ii) An $n \times n$ matrix A is diagonalisable if and only if $\mathbb{R}^n$ has a basis of eigenvectors of A if and only if the sum of dimension of eigenspaces of A is n if and only if the algebraic and geometric multiplicities of eigenvalues of A coincide.

(b) Orthogonal diagonalization

   (i) Orthogonal diagonalization of $n \times n$ real symmetric matrices.
   (ii) Application to real quadratic forms. Positive definite, semidefinite matrices. Classification in terms of principal minors. Classification of conics in $\mathbb{R}^2$ and quadric surfaces in $\mathbb{R}^3$.

Reference for Unit II:

(1) S. Kumaresan, Linear Algebra: A Geometric Approach.

(2) M. Artin. Algebra. Prentice Hall.

(3) T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.

(4) L. Smith, Linear Algebra, Springer.

Unit III. Groups and subgroups (15 Lectures)

(a) Definition and properties of a group. Abelian group. Order of a group, finite and infinite groups. Examples of groups including

   (i) $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$ under addition.
   (ii) $\mathbb{Q}^*(= \mathbb{Q} \setminus \{0\})$, $\mathbb{R}^*(= \mathbb{R} \setminus \{0\})$, $\mathbb{C}^*(= \mathbb{C} \setminus \{0\})$, $\mathbb{Q}^+$(= positive rational numbers) under multiplication.
   (iii) $\mathbb{Z}_n$, the set of residue classes modulo n under addition.
   (iv) $U(n)$, the group of prime residue classes modulo n under multiplication.
   (v) The symmetric group S_n.
   (vi) The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetrices of a regular polygon of n sides (for $n = 3, 4$).
   (vii) Klein 4-group.
   (viii) Matrix groups $\mathrm{M}_{m\times n}(\mathbb{R})$ under addition of matrices, $\mathrm{GL}_n(\mathbb{R})$, the set of invertible real matrices, under multiplication of matrices.

(b) Subgroups and Cyclic groups.

   (i) S^1 as subgroup of $\mathbb{C}$, μ_n the subgroup of n-th roots of unity.
   (ii) Cyclic groups (examples of $\mathbb{Z}$, $\mathbb{Z}_n$, and μ_n) and cyclic subgroups.
   (iii) The Center $Z(G)$ of a group G as a subgroup of G.
   (iv) Cosets, Lagrange's theorem.

(c) Group homomorphisms and isomorphisms. Examples and properties. Automorphisms of a group, inner automorphisms.

Reference for Unit III:

(1) I.N. Herstein, Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books

1. S.Kumaresan. Linear Algebra: A Geometric Approach, Prentice Hall of India Pvt Ltd, New Delhi.

2. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.

3. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.

4. M. Artin, Algebra, Prentice Hall of India, New Delhi.

5. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.

6. L. Smith, Linear Algebra, Springer.

7. Tom M. Apostol, Calculus Volume 2, Second edition, John Wiley, New York, 1969.

8. P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.

9. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.

10. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books

1. S. Lang, Introduction to Linear Algebra, Second edition, Springer Verlag, New York.

2. K. Hoffman and S. Kunze, Linear Algebra, Prentice Hall of India, New Delhi.

3. S. Adhikari. An Introduction to Commutative Algebra and Number theory. Narosa Publishing House.

4. T.W. Hungerford. Algebra. Springer.

5. D. Dummit, R. Foote. Abstract Algebra. John Wiley & Sons, Inc.

6. I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.

Course: TOPOLOGY OF METRIC SPACES I
Course Code: USMT503 /UAMT503

Unit I. Metric spaces (15 Lectures)

(a) (i) Metrics spaces: Definition, Examples, including $\mathbb{R}$ with usual distance, discrete metric space.

(ii) Normed linear spaces: Definition, the distance (metric) induced by the norm, translation invariance of the metric induced by the norm. Examples including

(1) $\mathbb{R}^n$ with sum norm $|| \, ||_1$, the Euclidean norm $|| \, ||_2$, and the sup norm $|| \, ||_\infty$.

(2) $C[a,b]$, the space of continuous real valued functions on $[a,b]$ with norms $|| \, ||_1$, $|| \, ||_2$, $|| \, ||_\infty$, where $||f||_1 = \int_a^b |f(t)|dt$, $||f||_2 = \left(\int_a^b |f(t)|^2 dt\right)^{\frac{1}{2}}$, $||f||_\infty = \sup\{|f(t)|, t \in [a,b]\}$.

(iii) Subspaces, product of two metric spaces.

(b) (i) Open ball and open set in a metric space (normed linear space) and subspace Hausdorff property. Interior of a set.

(ii) Structure of an open set in $\mathbb{R}$, namely any open set is a union of a countable family of pairwise disjoint intervals.

(iii) Equivalent metrics, equivalent norms.

(c) (i) Closed set in a metric space (as complement of an open set), limit point of a set (A point which has a non-empty intersection with each deleted neighbourhood of the point), isolated point. A closed set contains all its limit points.

(ii) Closed balls, closure of a set, boundary of a set in a metric space.

(iii) Distance of a point from a set, distance between two sets, diameter of a set in a metric space.

Reference for Unit I:

1. S. KUMARESAN, Topology of Metric spaces.

2. W. RUDIN, Principles of Mathematical Analysis.

Unit II. Sequences (15 Lectures)

(a) (i) Sequences in a metric space.

(ii) The characterization of limit points and closure points in terms of sequences.

(iii) Dense subsets in a metric space. Separability, $\mathbb{R}$ is separable.

(iv) Cauchy sequences and complete metric spaces. $\mathbb{R}^n$ with Euclidean metric is a complete metric space.

(b) Cantor's Intersection Theorem.

Reference for Unit II:

1. S. Kumaresan, Topology of Metric spaces.
2. W. Rudin, Principles of Mathematical Analysis.

Unit III. Continuity (15 Lectures)

$\varepsilon - \delta$ definition of continuity at a point of a function from one metric space to another.

(a) Characterization of continuity at a point in terms of sequences, open sets.

(b) Continuity of a function on a metric space. Characterization in terms of inverse image of open sets and closed sets.

(c) Algebra of continuous real valued functions.

(d) Uniform continuity in a metric space, definition and examples (emphasis on $\mathbb{R}$).

Reference for Unit III: S. Kumaresan, Topology of Metric spaces.

Course: Numerical Methods I (Elective A)
Course Code: USMT5A4 / UAMT5A4

Unit I. Transcendental equations (15 Lectures)

(a) Errors, type of errors - relative error, absolute error, round-off error, truncation error. Examples using Taylors series.

(b) Iteration methods based on first degree equation

   (i) The Newton-Raphson method

   (ii) Secant method

   (iii) Regula-Falsi method

(c) Iteration methods based on second degree equation - Muller method (problem to be asked only for one iteration)

(d) General iteration method - Fixed point iteration method.

(e) Rate of convergence of

   (i) The Newton-Raphson method

   (ii) Secant method

   (iii) Regula-Falsi method

Reference of Unit I:

(1) M.K. Jain, S.R.K.Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New age International publishers, Fourth Edition, 2003.

(2) B.S. Grewal, *Numerical Methods in Engineering and Science.* Khanna publishers.

Unit II. Polynomial and System of linear algebraic equations (15 Lectures)

(a) Polynomial equations

   (i) Sturm sequence

   (ii) Birge-vieta method

   (iii) Graeffe's roots squaring method

(b) Linear systems of equations

   (i) Direct methods - Triangularization method, Cholesky method

   (ii) Iteration methods - Jacobi iteration method.

Reference of Unit II:

(1) M.K. Jain, S.R.K.Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New age International publishers, Fourth Edition, 2003.

(2) B.S. Grewal, *Numerical Methods in Engineering and Science.* Khanna publishers.

Unit III. Eigenvalue problems.(15 Lectures)

Eigenvalues and eigenvectors

(i) Jacobi methods for symmetric matrices

(ii) Rutihauser method for arbitrary matrices

(iii) Power method

(iv) Inverse Power method

Reference of Unit III:

(1) M.K. Jain, S.R.K.Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New age International publishers, Fourth Edition, 2003.

(2) B.S. Grewal, *Numerical Methods in Engineering and Science*, Khanna publishers.

References:

(1) M.K. Jain, S.R.K.Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New age International publishers, Fourth Edition, 2003.

(2) B.S. Grewal, *Numerical Methods in Engineering and Science.* Khanna publishers.

(3) S.D. Comte and Carl de Boor, *Elementary Numerical analysis - An Algorithmic approach*, 3rd Edition., McGraw Hill, International Book Company, 1980.

(4) James B. Scarboraugh, *Numerical Mathematical Analysis*, Oxford and IBH Publishing Company, New Delhi.

(5) F.B. Hildebrand, *Introduction to Numerical Analysis*, McGraw Hill, New York, 1956.

Course: Number Theory and its applications I (Elective B)
Course Code: USMT5B4 / UAMT5B4

Unit I. Prime numbers and congruences (15 Lectures)

(a) (i) Review of divisibility.

(ii) Primes: Definition, The fundamental theorem of Arithmetic Distribution of primes (There are arbitrarily large gaps between consecutive primes).

(b) Congruences

(i) Definition and elementary properties, complete residue system modulo m. Reduced residue system modulo m, Euler's function ϕ.

(ii) Euler's generalization of Fermat's little Theorem, Fermat's little Theorem, Wilson's Theorem. The Chinese remainder Theorem.

(iii) With Congruences of degree 2 with prime modulii.

Unit II. Diophantine equations and their solutions (15 Lectures)

Diophantine equations and their solutions

(a) The linear equations $ax + by = c$.

(b) Representation of prime as a sum of two squares.

(c) The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions.

(d) The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions (x, y, z) with $xyz \neq 0$.

(e) Every positive integer n can be expressed as sum of squares of four integers, Universal quadratic forms $x^2 + y^2 + z^2 + t^2$.

Unit III. Quadratic Reciprocity (15 Lectures)

(a) Quadratic residues and Legendre Symbol. The Gaussian quadratic reciprocity law.

(b) The Jacobi Symbol and law of reciprocity for Jacobi Symbol.

(c) Special numbers; Fermat numbers; Mersene numbers; Perfect numbers, Amicable numbers.

Reference Books

1. I. Niven, H. Zuckerman and H. Montogomery. *Elementary number theory.* John Wiley & Sons. Inc.

2. David M. Burton. *An Introduction to the Theory of Numbers.* Tata McGraw Hill Edition

3. G. H. Hardy, and E.M. Wright, *An Introduction to the Theory of Numbers.* Low priced edition. The English Language Book Society and Oxford University Press, 1981.

4. Neville Robins. *Beginning Number Theory*, Narosa Publications.

5. S.D. Adhikari. *An introduction to Commutative Algebra and Number Theory.* Narosa Publishing House.

6. S. B. Malik. *Basic Number theory.* Vikas Publishing house.

7. N. Koblitz. *A course in Number theory and Crytopgraphy.* Springer.

8. M. Artin.*Algebra.* Prentice Hall.

9. K. Ireland, M. Rosen. *A classical introduction to Modern Number Theory.* Second edition, Springer Verlag.

10. William Stalling. *Cryptology and network security.*

Course: Graph Theory and Combinatorics I (Elective C)
Course Code: USMT5C4 / UAMT5C4

Unit I. Basics of Graph Theory (15 Lectures)

Review: Definitions and basic properties of

(i) Simple, multiple and directed graphs.

(ii) Degree of a vertex, walk, path, tree, cycle, complement of a graph, etc.

(a) Connected Graphs: Subgraphs, induced subgraphs - Definition and simple examples. Connected graphs, connected components, adjacency and incidence matrix of a graph. Results such as:

  (i) If $G(p, q)$ is self complementary graph, then $p \equiv 0, 1(\text{mod}4)$.

  (ii) For any graph G, either G or G^c is connected.

  (iii) Degree sequence - Havel Hakimi theorem.

(b) Trees- Definition of Tree, Cut vertices and cut edges, Spanning tree, Equivalent definitions of tree, Characterisations of trees such as

  (i) Any two vertices are connected by a unique path.

  (ii) The number of edges is one less than the number of vertices in a tree

Unit II. Spanning Tree (15 Lectures)

(a) If T is a spanning tree in a connected graph G and e is an edge of G that is not in T, then $T + e$ contains a unique cycle that contains the edge e.

(b) A rooted tree, binary tree, Huffman code (or prefix-free code), Hufman's Algorithm.

(c) Counting the number of spanning trees. Definitions of the operations $G - e$ and $G.e$ where e is any edge of G.

(d) If $\tau(G)$ denotes the number of spanning trees in a connected graph G, then, $\tau(G) = \tau(G - e) + \tau(G.e)$. Small examples, Cayley's theorem (with proof).

Unit III. Hamiltonian Graphs (15 Lectures)

(a) Hamiltonian graphs - Introduction and Basic definitions.

(b) If G is Hamiltonian graph, then $\omega(G - S) \leq |S|$ where S is any subset of vertex set V of G.

(c) Hamilton cycles in a cube graph.

(d) Dirac Result and Hamiltonian closure.

References:

1. Biggs Norman, Algebraic Graph Theory,Cambridge University Press.
2. Bondy J A, Murty U S R. Graph Theory with Applications,Macmillan Press.
3. Brualdi R A, Introductory Combinatorics, North Holland Cambridge Company. Cohen
4. D A, Basic Tecniques of Combinatorial Theory,John Wiley and sons. Tucker Allan
5. Applied Combinatoricds, John Wiley and sons.
6. Robin J. Wilson, "Introduction to Graph Theory", Longman Scientific & Technical.
7. Joan M. Aldous and Robin J. Wilson, "Graphs and Applications", Springer (indian Ed) West D B
8. Introduction to Graph Theory, Prentice Hall of India.

Course: Probability and Application to Financial Mathematics I (Elective D)
Course Code: USMT5D4 / UAMT5D4

Unit I. Probability as a Measure-Basics (15 Lectures)

(a) Modeling random experiments

(b) Uniform Probability measure

(c) Conditional Probability and Independence, Total Probability theorem, Baye's theorem.

(d) Fields and Finitely additive probability measure

(e) Sigma fields

  (i) σ field generated by a family of subsets of Ω.

  (ii) σ field of Borel sets.

(f) Upper limit (limit superior), lower limit(limit inferior) of a sequence of events.

Reference of Unit I: Chapter 1-5, Chapter 7 of Marek Capinski , Tomas Zastawniak, *Probability through Problems*, Springer, Indian Reprint 2008.

Unit II. Absolutely continuous Probability measure, Random Variables (15 Lectures)

(a) Countably additive probability measure (extending notion of probability measure from fields to σ fields).

(b) Borel Cantellia lemma

(c) Lebesgue measure and Lebesgue integral (definition and statement of properties only)

(d) Density function, Using density function to define a probability measure on the real line, Absolutely continuous probability measure

(e) Given a probability space Ω with a σ field and probability measure P, define a function from Ω to $\mathbb{R}$ to be a random variable.

Reference of Unit II: Chapter 6, Chapter 8(8.1-8.5) of Marek Capinski , Tomas Zastawniak, *Probability through Problems*, Springer, Indian Reprint 2008.

Unit III. Joint Distributions and Conditional Expectation (15 Lectures)

(a) Joint distribution of random variables X and Y as a probability measure on $\mathbb{R}^2$.

(b) Expectation and Variance of discrete and continuous random variables.

(c) Jensen's inequality.

(d) Conditional Expectation.

Reference of Unit III: Chapter 8(8.6-8.9), Chapter 9, Chapter 10(10.1 -10.4) of Marek Capinski , Tomas Zastawniak, *Probability through Problems*, Springer, Indian Reprint 2008.

Course: Practicals (Based on USMT501 / UAMT501 and USMT502 / UAMT502)
Course Code: USMTP05 / UAMTP05

Suggested Practicals based on USMT501 / UAMT501

1. Riemann Integration.
2. Fundamental Theorem of Calculus.
3. Double and Triple Integrals.
4. Fubini's theorem, Change of Variables Formula.
5. Pointwise and uniform convergence of sequences and series of functions.
6. Illustrations of continuity, differentiability, and integrability for pointwise and uniform convergence. Term by term differentiation and integration.
7. Miscellaneous Theoretical questions based on full USMT501/ UAMT501 .

Suggested Practicals based on USMT502 / UAMT502

1. Quotient spaces.
2. Orthogonal transformations,Isometries.
3. Diagonalization.
4. Orthogonal diagonalization.
5. Groups, Subgroups, Lagrange's Theorem, Cyclic groups and Groups of Symmetry.
6. Group homomorphisms, isomorphisms.
7. Miscellaneous Theoretical questions based on full USMT502 / UAMT502.

Course: Practicals (Based on USMT503 / UAMT503 and USMT5A4 / UAMT5A4 OR USMT5B4 / UAMT5B4 OR USMT5C4 / UAMT5C4 OR USMT5D4 / UAMT5D4)
Course Code: USMTP06 / UAMTP06

Suggested Practicals based on USMT503 / UAMT503

(1). Metric spaces and normed linear spaces. Examples.

(2) Open balls, open sets in metric spaces, subspaces and normed linear spaces.

(3) Limit points: (Limit points and closure points, closed balls, closed sets, closure of a set, boundary of a set, distance between two sets).

(4) Cauchy Sequences, completeness

(5) Continuity.

(6) Uniform continuity in a metric space.

(7) Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals based on USMT5A4 / UAMT5A4

(1) The Newton-Raphson method, Secant method, Regula-Falsi method.

(2) Fixed point iteration method, Muller method.

(3) Polynomial equations - Sturm sequence, Birge-vieta method, Graeffe's roots squaring method.

(4) Linear systems of equations - Triangularization method, Cholesky method, Jacobi iteration method.

(5) Eigenvalues and eigenvectors - Jacobi methods for symmetric matrices, Rutihauser method for arbitrary matrices

(6) Eigenvalues and eigenvectors - Power method, Inverse Power method.

(7) Miscellaneous Theoretical questions based on full paper.

The Practicals should be performed using non-programmable scientific calculator. (The use of programming language like C or Mathematical Software like Mathematica, MatLab, MuPAD may be encouraged).

Suggested Practicals based on USMT5B4 / UAMT5B4

(1) Primes, Fundamental theorem of Airthmetic.

(2) Congruences.

(3) Linear Diophantine equation.

(4) Pythagorean triples and sum of squares.

(5) The Gaussian quadratic reciprocity law.

(6) Jacobi symbols and law of reciprocity for Jacobi symbols.

(7) Miscellaneous Theoretical questions based on full paper.

Suggested Practicals based on USMT5C4 / UAMT5C4

(1) Degree sequence and matrix representation of graphs.

(2) Cut vertices and cut edges

(3) Tree, Spanning tree

(4) Vertex and edge connectivity

(5) Hufman code.

(6) Hamiltonian graphs, Hamilton cycles in a cube graph

(7) Miscellaneous Theoretical questions based on full paper.

Suggested Practicals based on USMT5D4 / UAMT5D4

(1) Modeling random experiments, uniform probability measure, fields

(2) Sigma field, Baye's theorem

(3) Countably additive probability measure, Density function

(4) Random variable

(5) Joint distribution, Expectation and Variance of a random variable

(6) Conditional expectation

(7) Miscellaneous Theoretical questions based on full paper.

Revised Syllabus in Mathematics
Credit Based Semester and Grading System
Third Year B. Sc. / B. A. 2013-14

Semester VI

COURSE: REAL ANALYSIS AND MULTIVARIATE CALCULUS II
COURSE CODE: USMT601 / UAMT601

Unit I. Differential Calculus (15 Lectures)

(a) Review of functions from $\mathbb{R}^n$ to $\mathbb{R}$ (scalar fields), Iterated limits.

(b) Limits and continuity of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$ (Vector fields)

(c) Basic results on limits and continuity of sum, difference, scalar multiples of vector fields.

(d) Continuity and components of vector fields.

(e) Derivative of a scalar field with respect to a vector.

(f) Direction derivatives and partial derivatives of scalar fields.

(g) Mean value theorem for derivatives of scalar fields.

Reference for Unit I:

(1) **Calculus, Vol. 2**, T. APOSTOL, John Wiley.

(2) **Calculus**. J. STEWART. Brooke/Cole Publishing Co.

Unit II. Differentiability (15 Lectures)

(a) Differentiability of a scalar field at a point (in terms of linear transformation).

Total derivative. Uniqueness of total derivative of a differentiable function at a point. (Simple examples of finding total derivative of functions such as $f(x, y) = x^2+y^2$, $f(x, y, z) = x+y+z$, may be taken). Differentiability at a point implies continuity, and existence of direction derivative at the point. The existence of continous partial derivatives in a neighbourhood of a point implies differentiability at the point.

(b) Gradient of a scalar field. Geometric properties of gradient, level sets and tangent planes.

(c) Chain rule for scalar fields.

(d) Higher order partial derivatives, mixed partial derivatives.

Sufficient condition for equality of mixed partial derivative.

Second order Taylor formula for scalar fields.

(e) Differentiability of vector fields.

(i) Definition of differentiability of a vector field at a point.
Differentiability of a vector field at a point implies continuity.

(ii) The chain rule for derivative of vector fields (statement only).

Reference for Unit II:

(1) **Calculus, Vol. 2**, T. Apostol, John Wiley.

(2) **Calculus**. J. Stewart. Brooke/Cole Publishing Co.

Unit III. Surface integrals (15 Lectures)

(a) (i) Parametric representation of a surface.
(ii) The fundamental vector product, definition and it being normal to the surface.
(iii) Area of a parametrized surface.

(b) (i) Surface integrals of scalar and vector fields (definition).
(ii) Independence of value of surface integral under change of parametric representation of the surface (statement only).
(iii) Stokes' theorem, (assuming general form of Green's theorem) Divergence theorem for a solid in 3-space bounded by an orientable closed surface for continuously differentiable vector fields.

Reference for Unit III:

(1) **Calculus. Vol. 2**, T. Apostol, John Wiley.

(2) **Calculus**. J. Stewart. Brooke/Cole Publishing Co.

Recommended books:

(1) Robert G. Bartle and Donald R. Sherbert. Introduction to Real Analysis, Second edition, John Wiley & Sons, INC.

(2) Richard G. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt. Ltd., New Delhi.

(3) Tom M. Apostol, Calculus Volume II, Second edition, John Wiley & Sons, New York.

(4) J. Stewart. Calculus. Third edition. Brooks/Cole Publishing Co.

(5) Berberian. Introduction to Real Analysis. Springer.

Additional Reference Books:

(1) J.E. Marsden and A.J. Tromba, Vector Calculus. Fifth Edition, http://bcs.whfreeman.com/marsdenvc5e/

(2) R. Courant and F. John, Introduction to Calculus and Analysis, Volume 2, Springer Verlag, New York.

(3) M.H. Protter and C.B. Morrey, Jr., Intermediate Calculus, Second edition, Springer Verlag, New York, 1996.

(4) D.V. Widder, Advanced Calculus, Second edition, Dover Pub., New York.

(5) Tom M. Apostol, Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.

(6) J. Stewart. Multivariable Calculus. Sixth edition. Brooks/Cole Publishing Co.

(7) George Cain and James Herod, Multivariable Calculus. E-book available at http://people.math.gatech.edu/ cain/notes/calculus.html

Course: Algebra II
Course Code: USMT602 / UAMT602

Unit I. Normal subgroups (15 Lectures)

(a) (i) Normal subgroups of a group. Definition and examples including center of a group.
 (ii) Quotient group.
 (iii) Alternating group A_n, cycles. Listing normal subgroups of A_4, S_3.

(b) First Isomorphism theorem (or Fundamental Theorem of homomorphisms of groups).

(c) Cayley's theorem.

(d) External direct product of a group. Properties of external direct products. Order of an element in a direct product, criterion for direct product to be cyclic.

(e) Classification of groups of order ≤ 5.

Reference for Unit I:

(1) I.N. Herstein. Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit II. Ring theory (15 Lectures)

(a) (i) Definition of a ring. (The definition should include the existence of a unity element.)
 (ii) Properties and examples of rings, including $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $M_n(\mathbb{R})$, $\mathbb{Q}[X]$, $\mathbb{R}[X]$, $\mathbb{C}[X]$, $\mathbb{Z}[i]$, $\mathbb{Z}[\sqrt{2}]$, $\mathbb{Z}[\sqrt{-5}]$, $\mathbb{Z}_n$.
 (iii) Commutative rings.
 (iv) Units in a ring. The multiplicative group of units of a ring.
 (v) Characteristic of a ring.
 (vi) Ring homomorphisms. First Isomorphism theorem of rings.
 (vii) Ideals in a ring, sum and product of ideals in a commutative ring.
 (viii) Quotient rings.

(b) Integral domains and fields. Definition and examples.
 (i) A finite integral domain is a field.
 (ii) Characteristic of an integral domain, and of a finite field.

(c) (i) Construction of quotient field of an integral domain (Emphasis on $\mathbb{Z}$, $\mathbb{Q}$).
 (ii) A field contains a subfield isomorphic to $\mathbb{Z}_p$ or $\mathbb{Q}$.

Reference for Unit II:

(1) M. Artin. Algebra.

(2) N.S. Gopalkrishnan. University Algebra.

(3) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit III. Factorization. (15 Lectures)

(a) Prime ideals and maximal ideals. Definition and examples. Characterization in terms of quotient rings.

(b) Polynomial rings. Irreducible polynomials over an integral domain. Unique Factorization Theorem for polynomials over a field.

(c) Divisibility in an integral domain, irreducible and prime elements, ideals generated by prime and irreducible elements.

(d) (i) Definition of a Euclidean domain (ED), Principal Ideal Domain (PID), Unique Factorization Domain (UFD). Examples of ED: $\mathbb{Z}$, $F[X]$, where F is a field, and $\mathbb{Z}[i]$.

(ii) An ED is a PID, a PID is a UFD.

(iii) Prime (irreducible) elements in $\mathbb{R}[X]$, $\mathbb{Q}[X]$, $\mathbb{Z}_p[X]$. Prime and maximal ideals in polynomial rings.

(iv) $\mathbb{Z}[X]$ is not a PID. $\mathbb{Z}[X]$ is a UFD (Statement only).

Reference for Unit III:

(1) M. Artin. Algebra.

(2) N.S. Gopalkrishnan. University Algebra.

(3) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books

1. S.Kumaresan. Linear Algebra: A Geometric Approach, Prentice Hall of India Pvt Ltd, New Delhi.
2. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
3. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
4. M. Artin, Algebra, Prentice Hall of India, New Delhi.
5. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.
6. L. Smith, Linear Algebra, Springer.
7. Tom M. Apostol, Calculus Volume 2, Second edition, John Wiley, New York, 1969.
8. P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
9. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
10. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books

1. S. Lang, Introduction to Linear Algebra, Second edition, Springer Verlag, New York.
2. K. Hoffman and S. Kunze, Linear Algebra, Prentice Hall of India, New Delhi.
3. S. Adhikari. An Introduction to Commutative Algebra and Number theory. Narosa Publishing House.
4. T.W. Hungerford. Algebra. Springer.
5. D. Dummit, R. Foote. Abstract Algebra. John Wiley & Sons, Inc.
6. I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.

Course: Topology of Metric Spaces II
Course Code: USMT603/ UAMT603

Unit I. Fourier Series (15 lectures)

(a) Fourier series of functions on $C[-\pi, \pi]$,

(b) Dirichlet kernel, Fejer kernel, Cesaro summability of Fourier series of functions on $C[-\pi, \pi]$

(c) Bessel's inequality and Pareseval's identity

(d) Convergence of the Fourier series in L^2 norm.

Reference for Unit I:

1. R. Goldberg. Methods of Real Analysis.
2. S. Kumaresan, Topology of Metric spaces.

Unit II. Compactness (15 lectures)

(a) Definition of a compact set in a metric space (as a set for which every open cover has a finite subcover). Examples, properties such as

- (i) continuous image of a compact set is compact.
- (ii) compact subsets are closed.
- (iii) a continuous function on a compact set is uniformly continuous.

(b) Characterization of compact sets in $\mathbb{R}^n$: The equivalent statements for a subset of $\mathbb{R}^n$ to be compact:

- (i) Heine-Borel property.
- (ii) Closed and boundedness property.
- (iii) Bolzano-Weierstrass property.
- (iv) Sequentially compactness property.

Reference for Unit II:

1. S. Kumaresan, Topology of Metric spaces.
2. W. Rudin, Principles of Mathematical Analysis.

Unit III. Connectedness (15 lectures)

(a)
- (i) Connected metric spaces. Definition and examples.
- (ii) Characterization of a connected space, namely a metric space X is connected if and only if every continuous function from X to $\{1, -1\}$ is a constant function.
- (iii) Connected subsets of a metric space, connected subsets of $\mathbb{R}$.
- (iv) A continous image of a connected set is connected.

(b)
- (i) Path connectedness in $\mathbb{R}^n$, definitions and examples.

(ii) A path connected subset of $\mathbb{R}^n$ is connected.

(iii) An example of a connected subset of $\mathbb{R}^n$ which is not path connected.

Reference for Unit III:

1. S. Kumaresan, Topology of Metric spaces.
2. W. Rudin, Principles of Mathematical Analysis.

Recommended Books

1. S. Kumaresan. Topology of Metric spaces.
2. R.G. Goldberg Methods of Real Analysis, Oxford and IBH Publishing House, New Delhi.
3. W. Rudin. Principles of Mathematical Analysis. McGraw Hill, Auckland, 1976.
4. P.K. Jain, K. Ahmed. Metric spaces. Narosa, New Delhi, 1996.
5. G.F. Simmons. Introduction to Topology and Modern Analysis. McGraw Hill, New York, 1963.

Additional Reference Books

1. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
2. E.T. Copson. Metric spaces. Universal Book Stall, New Delhi, 1996.
3. Sutherland. Topology.
4. D. Somasundaram, B. Choudhary. A first course in Mathematical Analysis. Narosa, New Delhi.
5. R. Bhatia. Fourier series. Texts and readings in Mathematics (TRIM series), HBA, India.

Course: Numerical Methods II (Elective A)
Course Code: USMT6A4 / UAMT6A4

Unit I. Interpolation (15 Lectures)

(a) Lagrange's Linear, quadratic and higher order Interpolation

(b) Iterated interpolation, Newton's divided difference interpolation

(c) Finite difference operators

(d) Interpolating polynomial using finite differences

Reference of Unit I: M.K.Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New age International publishers, Fourth Edition, 2003

Unit II. Interpolation and Differentiation (15 Lectures)

(a) Interpolation

- (i) Piecewise linear and quadratic interpolation.
- (ii) Bivariate interpolation - Newton's bivariate interpolation for equispaced points

(a) Numerical differentiation

- (i) Methods based on Interpolation (linear and quadratic), upper bound on the errors
- (ii) Partial differentiation

Reference of Unit II: M.K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New age International publishers, Fourth Edition, 2003.

Unit III. Numerical Integration (15 Lectures)

(a) Methods based on interpolation - Trapezoidal rule, Simpson's rule, error associated with these rules.

(b) Method based on undetermined coefficients - Gauss Legendre integration method (one point formula, two point formula)

(c) Composite integration methods - Trapezoidal rule, Simpson's rule

(d) Double integration - Trapezoidal method, simpson's method.

Reference of Unit III: M.K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New age International publishers, Fourth Edition, 2003.
References:

(1) M.K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New age International publishers, Fourth Edition, 2003.

(2) B.S. Grewal, *Numerical Methods in Engineering and Science*. Khanna publishers.

(3) S.D. Comte and Carl de Boor, *Elementary Numerical analysis - An Algorithmic approach*, 3rd Edition., McGraw Hill, International Book Company, 1980.

(4) James B. Scarboraugh, *Numerical Mathematical Analysis*, Oxford and IBH Publishing Company, New Delhi.

(5) F.B. Hildebrand, *Introduction to Numerical Analysis*, McGraw Hill, New York, 1956.

COURSE: NUMBER THEORY AND ITS APPLICATIONS II (ELECTIVE B)
COURSE CODE: USMT6B4 / UAMT6B4

Unit I. Continued Fractions (15 Lectures)

(a) Finite continued fractions

(b) (i) Infinite continued fractions and representatio of an irrational number by an infinite simple continued fraction.

(ii) Rational approximations to irrational numbers, Order of convergence, Best possible approximations.

(iii) Periodic continued fractions.

Unit II. Pell's equation, Arithmetic function and Special numbers (15 Lectures)

(a) Pell's equation $x^2 - dy^2 = n$, where d is not a square of an integer. Solutions of Pell's equation. (The proofs of convergence theorems to be omitted).

(b) Algebraic and transcendental numbers. The existence of transcendental numbers.

(c) Arithmetic functions of number theory: $d(n)$ (or $\tau(n)$), $\sigma(n)$ and their properties. $\mu(n)$ and the Möbius inversion formula.

(d) Special numbers: Fermats numbers, Perfect numbers, Amicable numbers. Pseudo primes, Carmichael numbers.

Unit III. Cryptography (15 Lectures)

(a) Basic notions such as encryption (enciphering) and decryption (deciphering).

Cryptosystems, symmetric key cryptography. Simple examples such as shift cipher, affine cipher, hill's cipher.

(b) Concept of Public Key Cryptosystem; RSA Algorithm.

Reference Books

1. I. NIVEN, H. ZUCKERMAN and H. MONTOGOMERY. *Elementary number theory.* John Wiley & Sons. Inc.

2. DAVID M. BURTON. *An Introduction to the Theory of Numbers.* Tata McGraw Hill Edition

3. G. H. HARDY, and E.M. WRIGHT, *An Introduction to the Theory of Numbers.* Low priced edition. The English Language Book Society and Oxford University Press, 1981.

4. NEVILLE ROBINS. *Beginning Number Theory*, Narosa Publications.

5. S.D. ADHIKARI. *An introduction to Commutative Algebra and Number Theory.* Narosa Publishing House.

6. S. B. MALIK. *Basic Number theory.* Vikas Publishing house.

7. N. KOBLITZ. *A course in Number theory and Crytopgraphy.* Springer.

8. M. Artin.*Algebra.* Prentice Hall.

9. K. Ireland, M. Rosen. *A classical introduction to Modern Number Theory.* Second edition, Springer Verlag.

10. William Stalling. *Cryptology and network security.*

Course: Graph Theory and Combinatorics II (Elective C)
Course Code: USMT6C4 /UAMT6C4

Unit I. Colouring in a graph and Chromatic number (15 Lectures)

(a) Introduction to vertex and edge colouring, Line graph of a graph, Vertex and edge chromatic number of a graph, Computation of vertex and edge chromatic number of graphs, Brooks theorem (without proof), Vizing theorem(without proof).

(b) Planner graph; Euler formula, Non planarity of $K_{3,3}$, K_5, Five-Colour theorem (with proof), Four- Colour theorem (Statement only).

(c) Chromatic polynomials-Basic results and computation of chromaticn polynomial of some simple graphs such as trees, cycles, complete graphs,wheele graphs, etc.

Unit II. Flow theory (15 Lectures)

(a) Flow Theory; Flow, Cut, Max ?ow, Min cut, Max ?ow-Min cut theorem, Flow theorem(without proof), Problems.

(b) System of distinct representatives- Hall's theorem (with proof).

Unit III. Combinatorics (15 Lectures)

(a) Applications of Inclusion - Exclusion principle-Forbidden position problem, Rook polynomial.

(b) Catalan number-Triangulation of a polygon, parenthesizing the product,deriving formula for catalan number, C_n.

(c) Introduction to ordinary generating functions. Solving recurrence relations using generating functions technique

References:

1. Biggs Norman, Algebraic Graph Theory,Cambridge University Press.
2. Bondy J A, Murty U S R. Graph Theory with Applications,Macmillan Press.
3. Brualdi R A, Introductory Combinatorics, North Holland Cambridge Company. Cohen
4. D A, Basic Tecniques of Combinatorial Theory,John Wiley and sons. Tucker Allan
5. Applied Combinatoricds, John Wiley and sons.
6. Robin J. Wilson, "Introduction to Graph Theory", Longman Scientific & Technical.
7. Joan M. Aldous and Robin J. Wilson, "Graphs and Applications", Springer (indian Ed) West D B
8. Introduction to Graph Theory, Prentice Hall of India.

Course: Probability and Application to Financial Mathematics II (Elective D)
Course Code: USMT6D4 / UAMT6D4

Unit I. Limit Theorems in Probability, Financial Mathematics-Basics (15 Lectures)

(a) Limit theorems

  (i) Chebyshev Inequality
  (ii) Weak law of large numbers
  (iii) Strong law of large numbers (statement only)
  (iv) Central limit theorem (statement only)

(b) A simple Market Model (Basic Notions and Assumptions)

(c) Risk-free Assets-

  (i) Time Value of money
  (ii) Money Market

(d) Risky Assets-

  (i) Dynamics of stock price - Return, Expected Return
  (ii) Binomial Tree Model, Risk-Neutral probability

Reference of Unit I: Chapter 1-3 of Marek Capinski , Tomas Zastawniak, *Mathematics for Finance-An Introduction to Financial Engineering*, Springer Undergraduate Mathematics Series, 2003 International Edition.

Unit II. Forward and Futures Contract (15 Lectures)

(a) Discrete Time Market Models

  (i) Investment Strategies
  (ii) The principle of No Arbitrage
  (iii) Application to the Binomial Tree Model
  (iv) Fundamental theorem of Asset pricing (statement only)

(b) Forward and Futures Contract

  (i) Pricing Forwards using Arbitrage
  (ii) Hedging with Futures

Reference of Unit II: Chapter 4, Chapter 6 of Marek Capinski , Tomas Zastawniak, *Mathematics for Finance-An Introduction to Financial Engineering*, Springer Undergraduate Mathematics Series, 2003 International Edition.

Unit III. Options (15 Lectures)

(a) Options

(i) Call and Put options

(ii) European and American options

(iii) Asian Options

(iv) Put-Call parity

(v) Bounds on option prices

(vi) Black Scholes Formula

(b) Portfolio Management

(i) Expected Return on a portfolio

(ii) Risk of a portfolio

Reference of Unit III: Chapter 5(5.1-5.3), Chapter 7 of Marek Capinski , Tomas Zastawniak, *Mathematics for Finance-An Introduction to Financial Engineering*, Springer Undergraduate Mathematics Series, 2003 International Edition.

Course: Practicals (Based on USMT601 / UAMT601 and USMT602 / UAMT602)
Course Code: USMTP07 / UAMTP07

Suggested Practicals based on USMT601 / UAMT601 and USMT602 / UAMT602

1. Limits and continuity of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, $m \geq 1$.
2. Partial derivative, Directional derivatives.
3. Differentiability of scalar fields.
4. Differentiability of vector fields.
5. Parametrisation of surfaces, area of parametrised surfaces, Surface integrals.
6. Stokes' Theorem and Gauss' Divergence Theorem.
7. Miscellaneous Theoretical questions based on full paper.

Suggested Practicals based on USMT602 / UAMT602

8. Normal subgroups and quotient groups.
9. Cayley's Theorem and external direct product of groups.
10. Rings, Integral domains and fields.
11. Ideals, prime ideals and maximal ideals.
12. Ring homomorphism, isomorphism.
13. Euclidean Domain, Principal Ideal Domain and Unique Factorization Domain.
14. Miscellaneous Theoretical questions based on full paper.

Course: Practicals (Based on USMT603 / UAMT603 and USMT6A4 / UAMT6A4 OR USMT6B4 / UAMT6B4 OR USMT6C4 / UAMT6C4 OR USMT6D4 / UAMT6D4)
Course Code: USMTP08 / UAMTP08

Suggested Practicals based on USMT603 / UAMT603

(1) Fourier series

(2) Parseval's identity.

(3) Compact sets in a metric space, Compactness in $\mathbb{R}^n$ (emphasis on $\mathbb{R}$, $\mathbb{R}^2$). Properties.

(4) Continuous image of a compact set.

(5) Connectedness, Path connectedness.

(6) Continuous image of a connected set.

(7) Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals based on USMT6A4 / UAMT6A4

(1) Lagrange's Linear, quadratic and higher order Interpolation, Iterated interpolation, Newton's divided difference interpolation

(2) Finite difference operators, Interpolating polynomial using finite differences

(3) Piecewise linear and quadratic interpolation, Newton's bivariate interpolation for equispaced points

(4) Numerical differentiation based on Interpolation and upper bound on the errors

(5) Numerical Integration - Trapezoidal rule, Simpson's rule, error associated with these rules, Gauss Legendre integration method (one point formula, two point formula)

(6) Composite integration (Trapezoidal rule, Simpson's rule), double integration (Trapezoidal rule, Simpson's rule)

(7) Miscellaneous Theoretical questions based on full paper.

The Practicals should be performed using non-programmable scientific calculator. (The use of programming language like C or Mathematical Software like Mathematica, MatLab, MuPAD may be encouraged).

Suggested Practicals based on USMT6B4 / UAMT6B4

(1) Finite continued fractions.

(2) Infinite continued fractions.

(3) Pell's equations.

(4) Arithmatic functions of number theory, Special numbers.

(5) Cryptosytems (Private key).

(6) Public Key Cryptosystems. RSA Algorithm.

(7) Miscellaneous Theoretical questions based on full paper.

Suggested Practicals based on USMT6C4 / UAMT6C4

(1) Chromatic Numbers (Vertex, Edge),Line graphs

(2) Planner graph, Chromatic polynomial.

(3) Flow Theorem.,Hall's theorem

(4) Recurrence relation, Generating function.

(5) Catalan numbers.

(6) Rook Polynomial.

(7) Miscellaneous Theoretical questions based on full paper.

Suggested Practicals based on USMT6D4 / UAMT6D4

(1) Limit theorems of probability

(2) Time value of Money, Expected Return

(3) No Arbitrage Principle, Pricing Forwards

(4) Binomial Tree Model- One period, Multiperiod

(5) Options, Put Call Parity

(6) Bounds on option Prices, Black Scholes Formula, Return and Risk of a portfolio.

(7) Miscellaneous Theoretical questions based on full paper.

The scheme of examination for the revised courses in the subject of Mathematics at the Third Year B.A./B.Sc. will be as follows.

Scheme of Examination (Theory)

The performance of the learners shall be evaluated into two parts. The learners performance shall be assessed by Internal Assessment with 40% marks in the first part by conducting the Semester End Examinations with 60% marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

(a) Internal assessment 40%
Courses with Assignments (Mathematics)

Sr. No.	Evaluation type	Marks
1	One assignments	10
2	One class test	20
3	Active participation in routine class	05
4	Overall conduct as a responsible student	05

(b) External Theory examination 60%

1. Duration: - Theses examinations shall be of $2\frac{1}{2}$ Hours duration.
2. Theory Question Paper Pattern:-
   (a) There shall be four questions each of 15 marks.
   (b) On each unit there will be one question and the fourth one will be based on entire syllabus.
   (c) All questions shall be compulsory with internal choice within the questions.
   (d) Each question may be subdivided into sub-questions a, b, c, and the allocations of marks depend on the weightage of the topic.
   (e) Each question will be of 20 to 23 marks when marks of all the subquestions are added (including the options) in that question.

Questions		**Marks**
Q1	Based on Unit I	15
Q2	Based on Unit II	15
Q3	Based on Unit III	15
Q4	Based on Unit I, II, III	15
	Total Marks	60

Semester End Examinations Practicals
At the end of the semester, examination of three hours duration and 100 marks shall be held for each course as given below.

Practical course	**Part A**	**Part B**	**Marks out of**	**Duration**
USMTP05 UAMTP05	Questions from USMT501/UAMT501	Questions from USMT502/UAMT502	80	3 Hrs
USMTP06 UAMTP06	Questions from USMT503/UAMT503	Questions from USMT5A4/UAMT5A4 USMT5B4/UAMT5B4 USMT5C4/UAMT5C4 USMT5D4/UAMT5D4	80	3 Hrs
USMTP07 UAMTP07	Questions from USMT601/UAMT601	Questions from USMT602 / UAMT602	80	3 Hrs
USMTP08 UAMTP08	Questions from USMT603/UAMT603	Questions from USMT6A4 / UAMT6A4 USMT6B4/UAMT6B4 USMT6C4/UAMT6C4 USMT6D4/UAMT6D4	80	3 Hrs

1. Journals: 10 Marks
2. Viva 10: Marks.

Pattern of the practical question paper at the end of the semester for each course: Every paper will consist of two parts A and B . Every part will consist of two questions of 40 marks. Students to attempt one question from each part.

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