Paper I – Algebra II

<u>Group Theory:</u> Action of groups, Sylow's theorem, applications, groups of order p, p^2, pq, where p and q are prime numbers: groups of order = 15;

Permutation groups; Cyclic decomposition, Alternating group An and simplicity of An linear groups

Normal series, solvable and nilpotent groups, Jordan holder theorem for groups.

<u>Modules:</u> Definition of modules, submodules, factor modules, homomorphisms, isomorphism theorems, direct command, direct sum of modules, free modules and bases, generators and relations of modules

Noetherian and Artinian modules and rings, Hilbert basis theorem

Finitely generated modules over a PID, application to the theory of single linear transformation

<u>Field theory:</u> Examples of fields, algebraic and transcendental elements.

The degree of field extensions; construction with ruler and compass;

Symbolic adjunction of roots, finite fields, algebraically closed fields;

Splitting fields and normal extensions, separable extensions, fundamental theorem of Galois theory,

Solvability by radicals, Examples of Galois groups, Cyclotomic fields, Symmetric Functions. (Cubic solvable).

References

1. Gallian.J.A. : Contemporary Abstract Algebra, (Fourth Ed.) Narosa, 1999.
2. Artin, M. : Algebra, Prentice — Hall of India, 1994.
3. Bhattacharya P.B., Jain S.K., Nagpan: Basic Abstract Algebra (Ind Ed.) (Sp. Ed- for South Asia), Cambridge Univ. Press 1995
4. Dummit. D.S., Foote R.M. : Abstract Algebra, J. Wiley (Indian Ed) 2002

Paper 2 - Analysis II

Lebesgue theory: Construction of Lebesgue measure, Measurable functions, Definition of Lebesgue integral and comparison with Rienmann integral, Limit. Theorems, Integration for complex valued functions, change of variable formula, Fubini theorem.

Normed linear spaces, characterization of finite dimensional normal linear spaces;

LP and l^p spaces as examples of Banach spaces;

Hilbert spaces, Orthogonal Complements, Orthonormal Bases, Orthonormal Expansions.

L^2 theory: Fourier Expansion, Examples like Legendre, Hermite functions and the related differential equations.

Fourier Analysis: Discussion on the convergence of Fourier Series, Plancherel theorem, Application of Fourier series to solve partial Differential Equations (Dirichlet problem, Mixed problem for wave and heat equations).

References

1. Lieb and Loss, Analysis, Narosa, 1993
2. Richard Beals, Advanced Mathematical Analysis, Narosa, 1980
3. Walter Rodin : Real and Complex Analysis. TMH, I974.
4. G. Folland, Real Analysis, John Wiley, 1984

Paper 3 – Differential Geometry

Euclidean spaces, Cauchy Schwarz inequality, Lines, Planes and Hyperplanes in R^n , Isometries of R^n Classification of Isometries of R^n

Linear systems of Ordinary differential equations; homogeneous and non-homogeneous cases; Nonlinear systems, local theory, fundamental theorem of existence and uniqueness of a solution; Flow of a differential equation.

Surfaces in R^3; Tangent space to a surface in 33; Surfaces locally as Graphs and Level sets, Orientable surfaces, constrained Maxima and Minima.

Curves in R^3; Serret-Frenet Equations, Fundamental Theorem of Curves.

Alternating Forms on a finite dimensional vector space over R^3, Differentiable forms, Exterior Derivative of Differential forms, Generalized Stokes theorem with classical applications.

First and Second Fundamental forms of surfaces in R^3; Principal Curvature and directions, Mean-Curvature vector, Gaussian curvature of at surface in 33; Local isometries between surfaces, Gauss's Theorema Egregium.

References

1. MP. do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, 1976.
2. M. Spivak, Calculus on Manifolds, WA. Benjamin, 1965
3. B.O'Neill, Elementary differential Geometry, Academic Press. 1966
4. J. Thorpe, Elementary Topics in Differential Geometry, Springer-Verlag, 1979
5. M. Apostol, Mathematical Analysis. 2nd edition. Narosa, 1984
6. V. Guillemin and P. Pollack , Differential Topology, van Nostrand 1970

Elementary Number Theory

Review of divisibility, primes, Fundamental Theorem of Arithmetic.

Euclid's and Euler's theorem on infinitude of primes, estimates for primes, Bertrand's postulate.

Arithmetic Functions like □(n), μ(n), divisor functions, etc.

<u>Congruences</u>: Chinese Remainder Theorem, use of congruences for solving Diophantine Equations.

Power Residues, primitive roots, quadratic reciprocity law and applications.

Finite Fields, Polynomials over finite fields, quadratic forms, Chevalley's theorem, Hasse's theorem on cubics.

Two square theorem, four square theorem, Legendre theorem on quadratic forms.

p-adic numbers, Hensel's lemma, Statement of Hasse-Minkowski's theorem.

Continued fractions, application to best approximation, Brahmagupta/Pell's, Cakravala method

Diophantine approximation, Liouville's theorem, construction of transcendental numbers,

transcendence of *e*.

References

1. I Niven, H. S. Zuckerman, H.L. Montgomery, An Introduction to the Theory of Numbers, Fifth Edition, John Wiley and Sons,1991.
2. Hardy and Wright. Number Theory. Clarendon Press, 1954
3. Apostol, Analytic Number Theory, Narosa, 1980
4. Alan Baker, A concise Introduction to the Theory of Numbers, Cambridge University Press, 1990
5. N. Koblitz, *p*-adic Numbers and *p*-adic Analysis. Springer-Verlag. GTM, 58.1984
6. K. Ireland, M. Rosen, A Classical Introduction to Modern Number Theory, Springer-Ver1ag, 84, 1982

Graph Theory

<u>Basic Concepts:</u> Basic concepts, hypercube graph, (complete) multipartite graph, cliques and independent sets, regular graphs, lower bound on the maximum number of edges in a bipartite subgraph of a graph, Graphic degree sequences, Havel-Hakimi theorem, Erdos-Gallai conditions (proof of necessity only), Turan's theorem on maximum number of edges in a graph with no K_{r+1}

<u>Distances, Paths and Trees</u>: Trees and Forests, Spanning subgraphs, Acyc!ic-subgraph paths, distance between two vertices, characterizations of a tree, spanning trees, centre of graph, radius and diameter of a graph, number of distinct spanning trees in a complete graph, Kruskal's algorithm for finding a minimum weight spanning tree (with proof of correctness of the algorithm), Prim's algorithm (with proof of correctness), shortest path and Dijkstra's algorithm (with proof of correctness), Breadth first and depth first search methods, rooted and binary trees, Huffman's algorithm for prefix-free coding with minimum expected search length, Eulerian circuits and trails, Fleury's algorithm for constructing Eulerian trails

<u>Matchings and Factors :</u> Matching and examples of a matching in some familiar graphs, alternating and augmenting path, maximum matching and augmenting path, Hall's matching condition for bipartite graphs, matchings in a regular bipartite graph, vertex cover, edge cover, independence number and their connections, matchings in general graphs and Tutte's theorem for the existence of a 1-factor in a graph

<u>Connectivity :</u> vertex out and connectivity, minimum number of edges in a *k*-connected graph, an edge cut, edge connectivity, vertex connectivity and the minimum degree, definition of a bond and its characterization, definition of a block and its properties, characterizations of a 2-connected graph, Menger's theorem and its applications, network flow problems, augmenting paths and characterization of a maximum flow, Ford-Fulkerson labeling algorithm, Integraity theorem, the demand-supply theorem and its application to the proof of Gale-Ryser theorem on degree sequences of a biaprtite graph

<u>Graph Colouring :</u> proper colourings and chromatic number of a graph, Greedy colouring, color critical graphs and various bounds on the chromatic number, Brook's theorem, triangle free graphs with arbitrary chromatic numbers. colour critical graphs and edge connectivity, chromatic polynomials and their properties, recurrence relation

<u>Edges and Cycles :</u> chromatic index of a graph, 1-factorizations, Vizing's theorem (statement only), elementary characterizations of a line graph, Hamiltonian paths and cycles, necessary condition (in terms of toughness) for a graph to Hamiltonian, Dira and Ore sufficiency conditions. Hamiltonian closure, Chvatal's theorem, examples of non-Hamiltonian graphs with large vertex degrees

<u>Planar Graphs:</u> Embedding of a graph on a plane, non-planarity of K_5 and $K_{3,3,}$ Dual of a graph. outerplanar graphs, Euler's formula for a planar graph and classification of regular polyhedra,

characterization of a planar graph and Kuratowski's theorem [Statement Only], 5-colour theorem and statement of the 4-colour theorem

Ramsey theory : Ramsey theorems for two colours and bounds on the number R(p, q), Ramsey numbers for three and more colours with no monochromatic triangles, Erlos-Szekeres theorem for convex *n*-gon for a given set of points in general position in the plane.

References

1. DB. West, Introduction to Graph Theory, Prentice-Hall of India Pvt. Ltd. 1999
2. J.A. Bond and UR. Murty, Graph Theory with applications, North-Holland, New York 1976
3. KR. Parthasarathy, Basic Graph Theory, Tate McGraw-Hill Publishing Company Ltd. 1994
4. B. Bollobas,Graph Theory, Springer Graduate Texts in Mathematics, Number 63

Functional Analysis

Norms and Semi norms, Equivalent norms, Banach and Hilbert Spaces, Examples: finite - dimensional normed linear space, l_p and L_p, spaces for ($1 < p < \infty$), the space C([0,3)] of continuous functions, Subspaces and Quotient spaces.

Finite dimens, Riesz Lemma.

Linear functionals. Hthn-Banach theorem and its consequences, Dual spaces, duals of standard spaces, Reflexive spaces, annihilators.

Linear transformations, The space B*(X,* Y) of bounded linear operators on X to Y. Open Mapping theorem and Closed graph theoerm, Uniform Boundedness theorerm, Banach-Steinhaus theorem, Applications.

Adjoint of an Operator, Compact operators, Riesz-Schauder Theory, Spectrum of a compact operator.

Integral Equations, Fredholm equations, Fredholm alternative.

Locally convex spaces. Weak topologies, Alaogious's theorem,

Unbounded Operators, Closed operators, The differential operator as example.

References

1. Kreyzig, Introductory Functional Analysis, and applications, John Wiley and Sons (Asia), 2001
2. B.V. Limaye : Functional Analysit„ 2nd Ed., New Age International, 1996
3. J.B. Conway : A course in functional Analysis, Springer-Verlag, GTM 96, 1990
4. *G. K.* Pedersen: Analysis Now, Springer-Verlag, GTM 118, 1989