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Abstract

       We propose a novel interpretation of Piero Sraffa's
(1960) Standard commodity: The solution price vector of
his subsistence production system is invariant to an
equivalence transformation of the constants of the
problem only in the case of the Standard system.

Key Words: Absolute values, Price stability

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1. INTRODUCTION

The opening salvo in one of the most deceptively terse assaults on received economic theory in our times is fired early in the preface of Sraffa's *Production of Commodities by Means of Commodities* (1960). "No changes in output ... are considered, so that no question arises as to the variation or constancy of returns" (Sraffa, I960, p.v). Furthermore, the classical approach is contrasted with the '"marginal"' method which "requires attention to be focused on change" (Sraffa, 1960, p.v). Thereafter, Sraffa proceeds to construct input-output models in classical fashion. We present his model of "Production with Subsistence" in vector-matrix form in the next section. The literal translation is not to be found, expositors moving to the schemes of expanded reproduction of the following chapters. In a departure from Sraffa's procedure in the book, (and the general equilibrium next step anywhere), however, we do not pick a commodity as a numeraire. In addition, we traverse the second chapter with its variables and equations.

The theme of change is revisited in the opening lines of the chapter of interest, "The Standard commodity", in the form of price changes, They are price movements brought about by a change in distribution. Also, in the case of a price fluctuation, Sraffa's problem is to distinguish between a change brought about by a variation in the price of the commodity of interest and the measuring rod. Since we are concerned neither with

standards of value nor distribution, our question is: How, otherwise, and if at all, might prices change? Our answer lies in the equivalence transformation of the constants of the problem. We show that the solution price vector changes pari passu only in the case of the Standard system. The theorem is proved in the following section. The relationship with the existing literature is sun eyed in Section III. The implications for the critique of general equilibrium economics follow. A final section is a summary.

2. THH RESULTS

Sraffa's economy consists of commodities, a, b, ..., k, each of which is produced by a separate industry. The quantity of "c' produced annually is A; B the similar quantity of 'b' and so on. The quantities of 'a','b' ..., 'k' annually used by the industry that produces A is $A_a, B_a, \ldots, K_a$; and $A_b, B_b, \ldots, K_b$ the corresponding quantities used for producing B; etcetera. These numbers are known. The unknowns to be determined are p_a, $p_b, \ldots, p_k$, the prices of the commodities 'a', 'b' ...,'k' respectively.

$$\text{Let } A = \begin{Vmatrix} A_a & A_b & \cdots & A_k \\ B_a & B_b & \cdots & B_k \\ \vdots & \vdots & \vdots & \vdots \\ K_a & K_b & \cdots & K_k \end{Vmatrix} \quad P \begin{Vmatrix} p_a \\ p_b \\ \vdots \\ p_k \end{Vmatrix} \quad Q = \begin{Vmatrix} A & 0 & \cdots & 0 \\ 0 & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K \end{Vmatrix}$$

The conditions of production appear as follows, where A^T is the transpose matrix:

$$A^T p = Q p \qquad (1)$$

Since the system is assumed to produce just enough to maintain itself, the following condition has to be met by the given matrices:

$$W A^{\mathsf{T}} = W Q \tag{2}$$

where $W = \begin{Vmatrix} 1 & 1 & . & \cdots & 1 \\ 0 & 0 & & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & & \cdots & 0 \end{Vmatrix}$. Premultiplication of the transpose of the

coefficient matrix by

the matrix W serves the purpose of adding the columns of A^{T}. The operation may be described as a 'pseudo' left-elementary operation. A 'proper[1] left-elementary operation would involve an invertible matrix.

Sraffa disciples, studying his unpublished work recently made available, claim that the Master did not regard the solution of his "first equations" as trivial (de Vivo, 2003; Garegnani, 2005; Kurz & Salvadori, 2005). In fact, his objective was the accomplishment of the task the classical economists could not complete in live absence of the required mathematical technology. The elaboration of a theory of value on the basis of production viewed as a circular flow could now be addressed with the tool of simultaneous equations. As early as November 1927, he notes that it is necessary to be aware of the properties of the solution set. He observes that the system admits of an infinite number of solutions all of which, however, are proportional to each other. These proportions he terms absolute values.

Our theorem then is

Proposition /: The absolute values of(l) and (2) arc not invariant to an equivalence
transformation of the coefficient matrices.

Proof. We can express equation (2) alternatively as

$$AW^{-1} = QW^{T}$$

(3)

where *W* , now, is a 'pseudo' right-elementary matrix.
We transform the 'old' basis into a 'new' basis, the vectors
and matrices in the new basis being given by starred
values.In terms *of* the new coordinates, then, equation
(I) becomes

$$A^{\prime *}p^* = Q^*p^*$$

(4)

Now,

$$A^* = U^{-1}AU \text{ and } Q^* = U^{-1}QU$$

(5)

where U is the matrix transforming the old basis vectors into
the new. Substituting this equation in (3), we get

(6)

$$UA^*U^{-1}W^{1} = UQ^*U^{-1}W^{\Gamma}$$

The matrix $U^{-1}H^{-1}$ continues to be a 'pseudo' right-elementary matrix. Multiply both equations on the right by any arbitrary A'-dimensional non-singular matrix *S*. Thereby, $U^{-1}H^{-1}S$, now, is a A'-dimensional non-singular matrix. Denote it by V^{-1}. Matrices A^{*} and O^{*} are now "properly" equivalent. That is to say,

$$A^{*} = U^{-1}AV = U^{-1}QV = Q^{*} \quad \textbf{(7)}$$

However, this equivalence is different from the equivalence (similarity) in (5). Conversely, the solution price vector associated with the equation (7) is unlikely to be equal to the solution price vector of (5). Note the arbitrary nature of the matrix *S*.

 The problem of constructing a Standard commodity is to find a set of A- multipliers, $q_1, q_2, \ldots, q_k$, to solve the following "q-system" (Sraffa, 1960, p.24).

$$Aq = Qq \quad (8)$$

where the matrices *A* and *O* are as before. This equation replaces equation (2). The corollary to our earlier result is

Proposition 2: The solutions of the Standard system, equations (1) and (8), are invariant lo an equivalence transformation of the coefficient matrices.

Proof. The derivation of equation (4) goes through in the identical manner. In similar fashion we consider the linear operator *A** and the vector *q** in the new basis. The solution of (8) is now

$$A^* q^* = Q^* q^*$$

3.(DIS)CONNECTIONS WITH THE LITERATURE

Our reading of Sraffa diverges completely from the interpretation of his critics as well as some leading followers who propose that the Standard commodity is an expendable device.

Grounds for meeting can be found in the view that the Standard system was not constructed to validate the labour theory of value or anything in particular. Indeed, a distinction is sought between Marx's intentions in this regard and that of Sraffa (Gilibert, 2003). Marx was concerned with the physical properties among industries which were essential for the smooth development of capitalist production. Sraffa, on the other hand, saw in Marx's schemes of reproduction important repercussions for the process of price determination. Continuing along this trajectory of sympathy, we end with the congruence of our result with the comment of Sraffa's literary executor that the Standard commodity is one expression *of* a system *of'* coordinates in a space and serves the abstract task of clarifying the properties when changes in the coordinates are possible (Garegnani, 2000, p.58). Another account which maps, at most points, onto our discussion, except for the view that the Standard commodity is not a construct to be used for investigation, is the

essay by Nistico and Rodano (2005). According to them, the theory of prices is the '"hard core'" of Sraffa's analysis (Nistico and Rodano, 2005, p.474). In a model with given quantities, prices must be the sole object of analysis. The prices of the first chapter of Sraffa's book are '"necessary real costs'" which arc unavoidable if the economy is to be reproduced (Nistico and Rodano. p.475). In an endorsement of the section to follow, they suggest that Sraffa's treatise strikes at the soft core of one of the foundations of the general equilibrium model, the (in)stability of the process of groping towards equilibrium.

4. "...A CRITIQUE OF ECONOMIC THEORY

Defenders of the neoclassical canon like Hahn and Samuelson frequently protest that the economic theory against which Sraffa's contribution is a counterpoise is a straw man. A modern user of general equilibrium theory, goes the oft-repeated complaint, would find the terms and conditions of Sraffa's programme incomprehensible. The paper suggests, on the other hand, that Sraffa's riposte can be given a contemporaneity and forward look that it allegedly lacks. Specifically, the charge against the general equilibrium model that endures is the instability of the tatonnement process. Up until the present, the device of the auctioneer has been indispensable in the dynamics of the familiar model of decentralized exchange in which prices and quantities move towards market-clearing levels.

5. CONCLUSION

We proffer a 'minimally indecomposable' interpretation of Sraffa's classic. It turns out that the returns, all the same, are strictly increasing. The solution price vector in the case of a classical stationary system changes arbitrarily in response to an equivalence transformation of the givens of the problem. Only Sraffa's Standard system can deliver the required invariance. Since the 'higgling of the market' in neoclassical economics is, in general, unstable, absent the introduction of strong and unrealistic assumptions, the "critique of economic theory" can move, along these lines, beyond a "prelude". Work **is** already underway to extend the device *of'* the Standard system to capture the insights of Sraffa's book following the first chapter.

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